Joint Determination of Event Epicenter and Magnitude from Seismic Intensities

by Seongjun Park and Tae-Kyung Hong

Abstract Characteristic features of seismicity with long recurrence intervals can be deduced from analysis of historical earthquake records that inherently suffer from uncertainty in the event locations and magnitudes. A novel method to determine the event epicenters and magnitudes jointly from seismic intensities is proposed. The probability for a set of event epicenter and magnitude is assessed by accounting the fitness between the observed and reference seismic intensities, spatial-occurrence probability based on seismicity density distribution, and temporal-occurrence probability from the Gutenberg–Richter magnitude–frequency relationship. A set of event epicenter and magnitude yielding the peak probability is chosen. The validity of the method is tested for both synthetic and instrumental seismic-intensity data, confirming high accuracy. The method is applied effectively to historical events with written seismic damage records. It is found that the errors generally decrease with increasing number and azimuthal coverage of seismic-intensity data, and increase with epicentral distances. The method appears to be promising for historical earthquakes of which source properties are poorly known. The method is applicable for assessment of the properties of long-period seismicity, which is crucial for assessment of potential seismic hazards.

Introduction

Large events naturally have long recurrence intervals (Shimazaki and Nakata, 1980; Schwartz and Coppersmith, 1984; Pearethree and Calvo, 1987). It is crucial to consider long-time seismicity for proper assessment of potential seismic hazards (e.g., Kawasumi, 1951; Frankel, 1995; McGuire, 1995; Stirling et al., 1998; Stirling and Petersen, 2006; Miyazawa and Mori, 2009). Instrumental seismicity records are only available for a short time (since the introduction of modern seismometers), which is insufficient for representation of seismicity of long recurrence intervals. The preinstrumental seismicity should be reflected for proper seismic-hazard analysis. Seismic damages by historical earthquakes are described in historical literatures. The seismic damages can be converted to seismic intensities. The historical earthquake records may be useful to assess the long-term seismicity. However, they have inherent limitation in source parameters including epicenters and magnitudes.

Seismic damage is generally proportional to the event magnitude and decreases with distance (Howell and Schultz, 1975). Conventionally, the epicenters of historical earthquakes are determined to be the locations of the largest damages or the central locations of equidamage regions (e.g., Degasperi et al., 1991; Wang, 2004; Lee and Yang, 2006; Seo et al., 2010). The event magnitudes are estimated from empirical relationships between seismic intensities and magnitudes.

When a number of seismic intensities are available over a wide region, an isoseismal map can be used for the determination of the event epicenter (e.g., Sibol et al., 1987; Levret et al., 1994; Albarello et al., 1995; Termini et al., 2005). In this case, the magnitude is estimated from the size of isoseismal area. The accuracy of the estimated epicenters and magnitudes is highly dependent on the spatial coverage of observed seismic damages in such conventional methods. Also, the event magnitude interlocks with the event location. Recently, Houng and Hong (2013) proposed a probabilistic approach to determine the epicenters of historical earthquakes using the instrumental seismicity densities, allowing analysis of offshore events as well as inland events. This probabilistic approach is useful to determine the stochastic properties of overall historical seismicity. However, the approach inherently suffers from nonunique determination of source parameters of particular events.

In this study, we propose a novel probabilistic method to determine the source parameters of historical events uniquely from seismic intensities using the instrumental seismicity density and the instrumental Gutenberg–Richter magnitude–frequency relationship. The proposed method is verified by synthetic tests and instrumental-event data. The method is
finally applied to historical seismic-intensity data for the
determination of unknown source parameters of historical
events.

Theory

Reference Seismic-Intensity Attenuation Curve

The seismic damage is proportional to seismic energy (magnitude) radiated from the sources and decreases with
distance (e.g., Bakun and Wentworth, 1997; Sørensen et al.,
2009; Szeliga et al., 2010). In addition, the seismic damage is
controlled by source radiation pattern, seismic attenuation,
site amplification, and source strength that are associated
with medium properties and tectonic environment (e.g., Ho-
well and Schultz, 1975; Tilford et al., 1985; Sokolov, 2002).
It is observed that the seismic intensities attenuate differently
with distance by region (e.g., Howell and Schultz, 1975;
Bakun and Wentworth, 1997; Musson, 2005; Szeliga et al.,
2010).

The reference seismic-intensity variation can be ex-
pressed as a function of magnitude and distance, which is
given by

\[ I_{\text{ref}}(m, l, h) = c + am - \beta \ln(l^2 + h^2) - \gamma \sqrt{l^2 + h^2} \] (1)

(e.g., Musson, 2005; Szeliga et al., 2010), in which \( I_{\text{ref}} \) is the
seismic intensity in modified Mercalli intensity (MMI) scale,
\( m \) is the magnitude, \( l \) is the epicentral distance in kilometers,
\( h \) is the focal depth in kilometers, and \( c, \alpha, \beta, \) and \( \gamma \) are cal-
ibration constants for source strength and seismic-intensity
attenuation rate associated with geometric spreading, anelas-
tic absorption, and scattering (Howell and Schultz, 1975).

Gutenberg–Richter Magnitude–Frequency
Relationship

It is known that natural seismicity satisfies the
Gutenberg–Richter magnitude–frequency relationship:

\[ \log(N_m) = a - bm \] (2)

(Gutenberg and Richter, 1954), in which \( N_m \) is the number of
events with magnitudes greater than, equal to, or larger than
\( m \), and \( a \) and \( b \) are constants. The relationship presents that
the earthquake recurrence interval increases with the magni-
tude. Constant \( a \) calibrates the number of earthquakes,
whereas constant \( b \) controls the relative occurrence frequency
for magnitudes. The constants \( a \) and \( b \) are region dependent
(e.g., Aki, 1965; Tinti and Mulfarga, 1987; Seo et al., 2010;
Hong, Lee, and Houng, 2015; Hong, Park, and Houng, 2015).
The constants \( a \) and \( b \) are determined for earthquake cata-
logs ensuring the completeness (e.g., Aki, 1965; Wiemer
and Wyss, 2000).

Seismicity Density Distribution

Seismic activity is controlled by the stress field and
medium properties, constructing characteristic distribution of
seismicity. Large earthquakes generally occur in high seis-
micity regions, following the general seismicity density dis-
tribution (e.g., Kossobokov et al., 2001; Houng and Hong,
2013). Houng and Hong (2013) presented a high correlation
between current seismicity density distribution and past large
earthquakes in California. The seismicity density distribution
can be useful to constrain possible event locations.

The seismicity density function is calculated by smooth-
ing the spatial densities of seismicity with magnitudes
greater than the minimum magnitude ensuring the complete-
ness of earthquake catalog. The seismicity density at the \( i \)th
cell \( D_i \) is given by

\[ D_i = \sum_{j=1}^{N_c} n_j \times \exp \left( \frac{-l_{ij}^2}{2\sigma_c^2} \right) \]

(Houng and Hong, 2013), in which \( N_c \) is the number of cells
discretizing the medium, \( n_j (j = 1, 2, \ldots, N_c) \) is the number
of earthquakes in the \( j \)th cell, \( l_{ij} \) is the epicentral distance
between the \( i \)th cell and the \( j \)th cell, and \( \sigma_c \) controls the de-
cay rate of the Gaussian function that is set to be 20 km
(Houng and Hong, 2013).

Probabilistic Determination of Source Parameters

Observed seismic intensities are expected to follow the
reference seismic-intensity attenuation curve that is a func-
tion of distance and magnitude. This feature allows us to con-
strain the event location and magnitude from the fitness
between the observed seismic intensities and the reference
seismic-intensity attenuation curve. The magnitude can be
additionally constrained by the Gutenberg–Richter relation-
ship that controls the relative occurrence chances among
events with different magnitudes. Events with larger magni-
tudes have lower chances of occurrence. Also, the spatial dis-
tribution of seismicity densities provides relative probability
as event location.

A posterior probability function is defined to be a prod-
uct of likelihood function and prior probability function (e.g.,
Vaseghi, 2001). The likelihood function of event location
and magnitude is determined using the fitness between the
reference and observed seismic intensities. The magnitude-
dependent event occurrence rate is used as a prior proba-
bility function of event magnitude. Also, the seismicity density is
used as a prior probability function of event location. The
posterior probability function is a multiplication of the like-
lihood function for seismic-intensity curve fitness and the
prior probability functions of event magnitude and location.

The posterior probability function \( P_{i,m} \) as a function of
location \( i \) and magnitude \( m \) is given by

\[ P_{i,m} = L_{i,m} \times F_m \times C_i, \] (4)
Joint Determination of Event Epicenter and Magnitude from Seismic Intensities

in which \( L_{i,m} \) is a function estimating the fitness between the reference and observed seismic intensities, function \( F_m \) accounts for the relative occurrence chance of earthquake with magnitude \( m \), and function \( C_i \) presents the seismicity density at the \( i \)th cell.

The differences between the reference and observed seismic intensities are expected to follow a Gaussian distribution (e.g., Gómez Cuépera, 2006). The seismic-intensity fitness function \( L_{i,m} \) is represented to be

\[
L_{i,m} = \frac{E_{i,m}}{E_{\text{max}}},
\]

in which \( E_{i,m} \) is the fitness function, and \( E_{\text{max}} \) is the peak fitness value for all discrete sets of \( i \) and \( m \) (\( i = 1, 2, ..., N_e \), \( m = M_{\text{min}}, ..., M_{\text{max}} \)) in which \( M_{\text{min}} \) and \( M_{\text{max}} \) are the minimum and maximum magnitudes. Function \( E_{i,m} \) is given by

\[
E_{i,m} = \prod_{j=1}^{n} \exp \left[ -\frac{\left( I_{\text{ref}}(m, l_{ij}, h) - I_{\text{obs}}(m, l_{ij}, h) \right)^2}{2\sigma_i^2} \right],
\]

in which \( n \) is the number of observed seismic intensities, \( I_{\text{ref}}(m, l_{ij}, h) \) is the reference seismic intensity at site \( j \) for an event with magnitude \( m \) occurring at the \( i \)th cell, \( l_{ij} \) is the epicentral distance between cell \( i \) and site \( j \), \( h \) is the focal depth, \( I_{\text{obs}} \) is the observed seismic intensity at site \( j \), and \( \sigma_i \) is the standard deviation of the Gaussian function.

The relative earthquake occurrence frequency of an event with magnitude \( m \), \( F_m \), is determined according to the Gutenberg–Richter magnitude–frequency relationship:

\[
F_m = \frac{10^{-bm}}{10^{-bM_{\text{min}}}},
\]

in which \( b \) is the Gutenberg–Richter constant. The normalized seismicity density at cell \( i \), \( C_i \), is given by

\[
C_i = \frac{D_i}{D_{\text{max}}},
\]

(Houng and Hong, 2013), in which \( D_i \) is a smoothed seismicity density for cell \( i \) that is given in equation (3), and \( D_{\text{max}} \) is the peak seismicity density for all cells (\( i = 1, 2, ..., N_e \)).

Equation (4) can be restated to be

\[
P_{i,m} = \exp \left[ -\frac{\sum_{j=1}^{n} \left( I_{\text{ref}}(m, l_{ij}, h) - I_{\text{obs}}(m, l_{ij}, h) \right)^2 + 2\ln(10)\sigma_i^2 b m}{2\sigma_i^2} \right] \times G_i,
\]

in which \( G_i \) is a composite magnitude-independent function:

\[
G_i = \frac{D_i}{10^{-bM_{\text{min}}} \times E_{\text{max}} \times D_{\text{max}}}. \quad (10)
\]

Here the event magnitude \( M_i \) yielding the peak posterior probability for event location \( i \) satisfies

\[
\frac{\partial P_{i,m}}{\partial m}|_{m=M_i} = 0, \quad (11)
\]

which gives

\[
\frac{\partial}{\partial m} \left[ \sum_{j=1}^{n} \left( F_{ij}^m - I_{\text{obs}}^m \right)^2 + 2\ln(10)\sigma_i^2 b m \right] = 0. \quad (12)
\]

If \( F_{ij}^m \) is expressed as equation (1), \( M_i \) satisfying equation (12) can be calculated by

\[
M_i = \frac{\sum_{j=1}^{n} \left( I_{\text{obs}}^m - c + \beta \ln(l_{ij}^m + h^2) + \gamma \sqrt{l_{ij}^m + h^2} \right)}{n \times \alpha} - \frac{\ln(10) \times b \times \sigma_i^2}{n \times \alpha^2}. \quad (13)
\]

The location \( i \) yielding the peak posterior probability with magnitude of \( M_i \) is selected as the optimal event location. Here, the event magnitude is determined subsequently. However, it is noteworthy that the likelihood function \( L_{i,m} \) varies according to the prior probability function \( F_m \). Thus, the magnitude \( M_i \) is expected to be deviated from the true optimal value, \( M_i^{op} \):

\[
M_i^{op} = M_i - \delta M, \quad (14)
\]

in which \( \delta M \) is the magnitude difference that is defined to be

\[
\delta M = M_i^{ex}(M^\alpha, i) - M^\alpha \quad (15)
\]

(Vaseghi, 2001), in which \( M_i^{ex}(M^\alpha, i) \) is the expected magnitude estimate for an event with true magnitude of \( M^\alpha \) and epicenter of \( i \). Here, from equation (13), \( M_i^{ex} \) can be calculated by

\[
M_i^{ex}(M^\alpha, i) = \frac{\sum_{j=1}^{n} \left( F_{ij}^m - c + \beta \ln(l_{ij}^m + h^2) + \gamma \sqrt{l_{ij}^m + h^2} \right)}{n \times \alpha} - \frac{\ln(10) \times b \times \sigma_i^2}{n \times \alpha^2}, \quad (16)
\]

in which \( F_{ij}^m(M^\alpha, i) \) is the expected seismic-intensity estimate at site \( j \) for an event with magnitude \( M^\alpha \) and epicenter \( i \). Also, the expected seismic-intensity estimate corresponds to \( F_{ij}^m(M^\alpha, l_{ij}, h) \), yielding the magnitude difference, \( \delta M \), to be

\[
\delta M = \frac{-\ln(10) \times b \times \sigma_i^2}{n \times \alpha^2}. \quad (17)
\]

Data and Seismicity Properties

The Korean Peninsula is a unique region with dense seismic monitoring networks and well-preserved historical earthquake records allowing us to test a new method for both instrumental
and historical earthquake data. We analyze 1119 instrumentally recorded earthquakes during 1978–2013 of which source information is collected from the Korea Meteorological Administration (KMA) and the Korea Institute of Geoscience and Mineral Resources (KIGAM) (Fig. 1). The event magnitudes are $M_L$ 1.7–5.3 (Fig. 2). The focal depths are shallower than 42 km, and the average focal depth is 7.3 km (Fig. 2).


Historical seismic damage records were originally collected from historical literatures including Samguksagi, Koryosa, and Choseonwangjosillog. The seismic intensities for the historical seismic damages are available from previous studies (e.g., Lee and Yang, 2006; KMA, 2012a). The number of reported historical earthquakes is 2161.

The Korean Peninsula is located in the eastern margin of the Eurasian plate that is adjacent to the Pacific plate and Philippine sea plate (Fig. 1). The plate collision geometry constructs an east-northeast–west-southwest directional compression field in and around the Korean Peninsula (Choi et al., 2012). The Korean Peninsula is located in a stable intraplate environment and is composed of complex geological and tectonic structures (Chough et al., 2000; Hong and Choi, 2012).

The reference seismic-intensity attenuation function for the Korean Peninsula is given by

$$I_{ref} = -0.998 + 1.72m - 0.322 \ln(l^2 + h^2) - 0.00608 \sqrt{l^2 + h^2}$$

(Park and Hong, 2014). The differences between the observed seismic intensities and the reference seismic-intensity

Figure 1. (a) Map of tectonic setting around the Korean Peninsula. The plate boundaries are marked with thick solid lines. The study area is marked with a rectangular box. (b) An enlarged map around the Korean Peninsula with seismicity during 1978–2013. The events with reported seismic intensities are marked with filled circles. The color version of this figure is available only in the electronic edition.

Figure 2. Distribution of (a) magnitudes and (b) focal depths of the instrumentally recorded earthquakes in Figure 1. The magnitudes range between 1.7 and 5.3. The focal depths of most events are less than 21 km. The average focal depth is 7.3 km.
The attenuation curve presents a normal distribution with a standard deviation of 0.65 in MMI unit. The threshold magnitude, $M_{\text{min}}$, ensuring the completeness of event catalog is determined to be 2.5 (Hong, Lee, and Houng, 2015; Hong, Park, and Houng, 2015). Also, the Gutenberg–Richter $b$-value is found to be around 0.92 (Hong, Lee, and Houng, 2015; Hong, Park, and Houng, 2015). The seismic density distribution is obtained from instrumentally recorded data for the region in latitudes between 32° and 42° N and longitudes between 122° and 132° E that is discretized into cells with a uniform size of $0.05° \times 0.05°$. The number of cells $N_c$ is 40,000. Earthquakes with magnitudes equal to or greater than the threshold magnitude ($M_{\text{min}} = 2.5$) are analyzed. The seismicity densities are observed to be high in the northwestern and southern Korean Peninsula, and relatively low in the central and northeastern Korean Peninsula. The seismicity is high in the regions off the west and southeast coasts. The color version of this figure is available only in the electronic edition.

Figure 3. Seismicity densities of instrumentally recorded earthquakes around the Korean Peninsula. The seismicity densities are high in the northwestern and southern Korean Peninsula, and relatively low in the central and northeastern Korean Peninsula. The seismicity is high in the regions off the west and southeast coasts. The color version of this figure is available only in the electronic edition.

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Validation Tests for Various Conditions

The azimuthal coverage, epicentral distances to stations, and number of observations are generally dependent on the physical and monitoring environment. The proposed method is tested with synthetic data for verification in various conditions. Synthetic data sets of seismic intensities for various configurations of observation positions are prepared. The observation points for each configuration are set to be placed evenly over the arc with a radius of the epicenter distance (Fig. 4). We consider the cases with the number of observation points of 1, 2, 3, 5, 10, and 20. Spatial distribution of observation points with azimuthal coverages of $10°$, $30°$, $60°$, $120°$, and $240°$ is considered. We consider epicentral distances of 10, 20, 50, 100, and 200 km for each case. We constitute 125 different configurations of observation positions from various combinations of azimuthal coverages, epicentral distances, and numbers of seismic intensities. The influences of azimuthal coverage, epicentral distance, and number of observations on the determination of magnitudes and epicenters are examined.

The event magnitude is set to be 5.0. The focal depth is set to be 7.3 km, considering the average focal depth of earthquakes in the Korean Peninsula. Synthetic seismic intensities are calculated from equation (18) with addition of Gaussian random errors of which standard deviation is set to be 0.65 in MMI unit, considering the residuals between the reference and observed seismic intensities (Park and Hong, 2014). The synthetic intensities are rounded off to be integers in MMI scale. We produce 100 synthetic seismic-intensity data sets for each observation configuration. A uniform seismicity density with the Gutenberg–Richter $b$-value of 0.92 is assumed over the medium. The medium is discretized into $1 \text{ km} \times 1 \text{ km}$ cells. We search the locations and magnitudes of peak probabilities. The apparent deviation of magnitudes is corrected using equation (15).

The estimated locations and magnitudes are compared with input parameters (Fig. 5). It is found that the accuracy of estimated parameters generally increases with the number of seismic intensities ($N_i$). Also, the error of estimated parameters is proportional to the distance. We find that the errors of estimated parameters decrease with the central angle of arc. The errors reach zero in cases with central angles greater than $120°$. The parameters are poorly estimated particularly with seismic-intensity data from stations on small azimuthal arcs in large distances.

Application to Synthetic Earthquakes

The proposed method is tested for imaginary earthquakes in the seismological environment of the Korean Peninsula. We consider earthquakes that are distributed randomly in the region with latitudes between 32° and 42° N and longitudes between 122° and 132° E. The actual instrumental seismicity density distribution over the peninsula is considered for the synthetic test. We consider imaginary seismic events with magnitudes equal to or greater than 3.0. The event magnitudes satisfying the Gutenberg–Richter magnitude–frequency relationship with $b$-value of 0.92 are selected randomly. The focal depth is set to be 7.3 km. The potential
locations of stations are on the inland region at every 0.01° in longitude and latitude (Fig. 6).

We generate 1000 imaginary events of which epicenters and magnitudes are chosen, considering the seismicity density distribution and Gutenberg–Richter magnitude–frequency relationship. The synthetic seismic intensities are produced from the reference seismic-intensity attenuation curve. The observation points of seismic intensities are chosen randomly. The selection chance of observation point is designed to be inversely proportional to its epicentral distance with an idea that a near location generally has a bigger seismic damage than a far location. We compose the synthetic data sets with \( N = 10, 2, 3, \ldots, 19 \), and 20 seismic intensities for each event. The seismic intensities are equal to or greater than MMI I.

The study region is discretized by \( 0.05° \times 0.05° \) cells. The probability of each discrete position is estimated. The location with the highest probability is determined to be the event epicenter. It is observed that the errors in event magnitudes and epicenters are highly dependent on the numbers of seismic intensities (Figs. 7, 8). The event magnitudes are generally underestimated when a single seismic-intensity observation is available. The average error in magnitude estimates for the inversions with single seismic intensities is \( -0.80 \) in magnitude unit, and the standard deviation is 0.64 in magnitude unit (Table 1).

On the other hand, the mean value and standard deviation of magnitude-estimate errors for inversions based on two seismic-intensity data are \(-0.26\) and \(0.49\) in magnitude unit, respectively, presenting significantly increased accuracy compared with inversions based on single seismic-intensity data. The accuracy of event magnitude increases with the number of seismic-intensity data. It is observed that inversions with 4 or more seismic-intensity data produce reasonable magnitude estimates with mean errors less than or equal to 0.1 in magnitude unit.
Figure 5. Variation of errors in (a) magnitudes and (b) locations as a function of epicentral distance of seismic-intensity observation point for various combinations of numbers of seismic-intensity data ($N_I = 1, 2, 3, 5, 10, 20$) and azimuthal ranges ($\theta = 10^\circ, 30^\circ, 60^\circ, 120^\circ, 240^\circ$) in synthetic experiments. The errors generally decrease with the number of seismic-intensity data and azimuthal coverage, and increase with distance. The color version of this figure is available only in the electronic edition.
The errors of epicenter estimates are populated around zero, suggesting reasonable determination of epicenters (Fig. 8). The 90% of epicenter estimates from the inversions based on single seismic-intensity data have the errors less than 153 km, while those from the inversions based on 2–5 seismic-intensity data display the errors less than 104–134 km. It is observed that the location errors of epicenters generally decrease with the number of seismic intensities.

It is noteworthy that the accuracy of epicenter estimates is dependent on the source environment. The magnitudes and epicenters of inland events are better determined than those of offshore events due to the azimuthal coverage of stations (Fig. 7). Also, the accuracy of inverted source parameters generally increases with the number of seismic-intensity data implemented for inversion. The offshore events naturally suffer from poor location constraints due to limited azimuthal coverage of stations and large epicentral distances. The source parameters of offshore events at large distances are poorly constrained even by the inversions based on large numbers of seismic-intensity data.

Application to Instrumental Earthquakes

The proposed method is applied to instrumentally recorded earthquakes with reported seismic intensities (Fig. 9). We analyze 55 instrumentally recorded earthquakes with magnitudes greater than or equal to 3.0 during 2001–2013. Examples of spatial distributions of seismic intensities are presented in Figure 9. The number of reported seismic intensities generally decreases with epicentral distance. The observation points of seismic intensities are populated near the epicenters for inland events (e.g., the 20 January 2007 M_{L} 4.8 earthquake and the 29 October 2008 M_{L} 3.4 earthquake), while around the coastal regions nearest to the epicenters for offshore events (e.g., the 31 May 2008 M_{L} 4.2 earthquake).

The epicenters and magnitudes of inland events are well determined from the inversion, while those of far offshore events are poorly constrained (Fig. 9d). Also, we observe that the errors of the inverted source parameters generally decrease with number of seismic-intensity data (Fig. 10). The errors observed in instrumental earthquakes are consistent with those observed in synthetic earthquakes, supporting the validity of the method.

Application to Historical Earthquakes

The method is applied to historical earthquakes of which source parameters were poorly constrained. Seismic damage records of two historical earthquakes are analyzed (Table 2). Seismic intensities are assigned to the seismic damage records (KMA, 2012a). The number of seismic-intensity data for the 20 July 1594 earthquake is 12, and the largest seismic intensity is determined to be MMI VIII. We also collect 11 seismic intensities varying up to MMI VI for the 2 November 1692 earthquake. The seismic intensities for the inland event (the 20 July 1594 earthquake) generally decrease with dis-
Figure 7. Determination of magnitudes and epicenters of 1000 synthetic events for inversions based on (a) 3, (b) 5, (c) 10, and (d) 20 seismic-intensity data ($N_I = 3, 5, 10, 20$). The determined locations of epicenters (open circles) are connected to the true locations (open triangles) by lines with presentation of magnitude errors ($\Delta M$). The magnitude and location errors generally decrease with the number of seismic-intensity data. Offshore events generally display larger errors than inland events due to limited azimuthal coverage and relatively large distances to locations of seismic intensities. The color version of this figure is available only in the electronic edition.

Figure 8. (a) Variation of magnitude errors ($M^{\text{est}} - M^{\text{tr}}$) as a function of the number of seismic-intensity data for a synthetic earthquake. The mean values and standard deviations are marked with bars. (b) Variation of location errors as a function of the number of seismic-intensity data. The 90% confidence range of the location errors is marked with a solid line. The magnitude and location errors generally decrease with the number of seismic-intensity data. The color version of this figure is available only in the electronic edition.
Based on conventional approach (Lee and Yang, 2006). In this study are compared with those from a previous study (Lee et al., 2011; Hough, 2014). The proposed method in this study may be suited for any regions with temporally stationary background seismicity. The method does not require including past large earthquakes in the reference seismicity records that is constructed based on the number of small-sized earthquakes. It was proved that the locations of past large earthquakes can be determined well based on the reference seismicity densities of small earthquakes (Houng and Hong, 2013). The observations suggest that the performance of the method may be hardly affected by the migration of large events. We expect that the proposed method may enable us to assess the seismicity properties of long periods that are crucial for assessment of potential seismic hazards.

### Table 1
Variation of Errors in Magnitudes and Epicenters with Change of the Number of Seismic-Intensity Data ($N_I$).

<table>
<thead>
<tr>
<th>$N_I$</th>
<th>$\Delta M$</th>
<th>$\sigma(\Delta M)$</th>
<th>$\Delta D$ (km)</th>
<th>$\sigma(\Delta D)$ (km)</th>
<th>$\Delta D_{90}$ (km)</th>
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The mean values $\Delta M$ and standard deviations $\sigma(\Delta M)$ of magnitude errors in magnitude unit, and the mean values $\Delta D$ and standard deviations $\sigma(\Delta D)$ of epicenter errors in kilometers along with the 90% confidence range $\Delta D_{90}$ are presented.
Figure 9. Determination of magnitudes and locations of three instrumentally recorded earthquakes: (a) the 20 January 2007 $M_L 4.8$ earthquake, (b) the 29 October 2008 $M_L 3.4$ earthquake, and (c) the 31 May 2008 $M_L 4.2$ earthquake. The seismic intensities are denoted with filled squares. The estimated magnitudes are close to the reported magnitudes, and the determined locations (filled stars) are placed near to the reported locations (filled circles). The spatial distribution of estimated probabilities is presented by contour lines. (d) The errors of estimated magnitudes and locations of all analyzed instrumentally recorded events are presented. The source parameters of inland events are determined well, while those of offshore events display large deviations. The color version of this figure is available only in the electronic edition.

Figure 10. (a) Variation of differences between estimated and reported magnitudes ($M^{est} - M^{rep}$) as a function of the number of seismic-intensity data. The mean values and standard deviations of the differences are denoted with bars. The 90% confidence range of errors from the synthetic tests is marked with solid lines. (b) Variation of differences between estimated and reported locations as a function of the number of seismic-intensity data. The 90% confidence range of location errors from the synthetic tests is presented (solid line). The observed differences of magnitudes and locations for instrumentally observed earthquakes are consistent with those for synthetic events. The color version of this figure is available only in the electronic edition.
Data and Resources


Acknowledgments

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References


Table 2

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<td>V</td>
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<td></td>
<td>III</td>
<td>10</td>
<td>Yangju (37.79°, 127.05°), Paju (37.76°, 126.78°), Icheon (37.27°, 127.43°), Jiye (37.47°, 127.64°), Yangpyeong (37.49°, 127.49°), Gongju (36.45°, 127.12°), Suncheon (34.95°, 127.49°), Mijang (35.42°, 126.56°), Uiseong (36.36°, 128.69°), Gangneung (37.75°, 128.88°)</td>
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</table>

MMI, modified Mercalli intensity.


