Modelling seismic waves in strongly heterogeneous media using a wavelet-based method

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ABSTRACT

Most numerical techniques for modelling seismic wave propagation encounter significant difficulties when confronted with media with strong heterogeneity. However, a wavelet-based approach can provide high accuracy and stability of spatial differentiation even in highly perturbed media. The wavelet-based method therefore allows the treatment of localized zones of strong contrast such as media with a fluid-filled crack.

The accuracy of the method makes it possible to consider seismic waves in a medium with a weak systematic structure such as subduction zones, where slabs exhibit mild-velocity contrast to the background and therefore there can be significant interface waves on the surfaces of the slabs when the source is close to the slab.

The wavelet-based method also allows an accurate treatment of the scattering effect of short-scale heterogeneity, as encountered in the crust. The results indicate that conventional finite difference methods are likely to overestimate scattering attenuation.

♠ *This poster is based on recent work [1–4] on the wavelet-based method.*

1 What do we need to consider for modelling in complex media with dynamic sources?

- can correct numerical responses be obtained in highly heterogeneous media?
- can complexity in source regions (e.g., heterogeneities, geometry) be treated?
- can realistic dynamic source (e.g., rupture propagation) be considered?

2 Wavelet-based method (WBM)

2.1 Numerical formulation

The 2-D *P-SV* elastic wave equation system including a body forces term (**f**):

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial j} + f_i, \quad (i, j = x, z). \tag{1}$$

where **u** is the displacement vector and σ is the stress tensor. Eq. (1) can be written in the form of first-order differential equation in time with a vector unknown **U**:

$$\partial_t \mathbf{U} = \mathbf{L}\mathbf{U} + \mathbf{N}, \tag{2}$$

where **U** is $(u_x, v_x, u_z, v_z)^t$, the operator matrix **L** including absorbing boundary conditions is given by

$$\mathbf{L} = \begin{pmatrix} 0 & I & 0 & 0 \\ \mathcal{L}_{xx} & -2Q_x & \mathcal{L}_{xz} & 0 \\ 0 & 0 & 0 & I \\ \mathcal{L}_{zx} & 0 & \mathcal{L}_{zz} & -2Q_z \end{pmatrix}, \tag{3}$$

and **N** is composed of body forces, $(0, f_x/\rho, 0, f_z/\rho)^t$. When additional boundary conditions are considered, such as traction-free conditions on a free surface or inside a medium (e.g., medium with a cavity), these conditions can be represented via equivalent forces using the stress values on the boundaries, and are added to body force components in **N**.

With a semigroup approach, the discrete time solution of (2) is given by

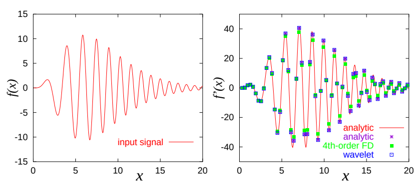
$$\begin{aligned} \mathbf{U}_{n+1} &= \mathbf{U}_n + \delta t \mathbf{L}\mathbf{U}_n + \frac{\delta t^2}{2}\mathbf{L}^2\mathbf{U}_n + \cdots + \frac{\delta t^m}{m!}\mathbf{L}^m\mathbf{U}_n \\ &\quad + \delta t \mathbf{N}_n + \frac{\delta t^2}{2}\mathbf{L}\mathbf{N}_n + \frac{\delta t^3}{6}\mathbf{L}^2\mathbf{N}_n + \cdots + \frac{\delta t^{m+1}}{(m+1)!}\mathbf{L}^m\mathbf{N}_n, \end{aligned} \tag{4}$$

where δt is a discrete time step, $\mathbf{U}_n$ is the displacement-velocity vector at discrete time t_n, $\mathbf{N}_n$ is a vector for forcing terms, and m controls the truncation order in the discrete time solution.

♠ *The procedure of numerical differentiation using wavelets is described in detail in [1, 2, 5], and the WBM is validated through comparisions with analytic solutions for some simple problems [1, 2].*

3 WBM in random heterogeneous media

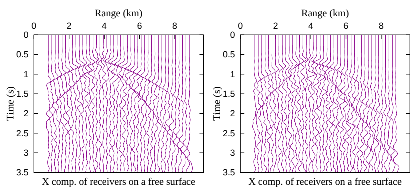
3.1 Accuracy test on numerical differentiation



- left: highly varying input signal regarded as the variation of physical parameters in highly perturbed media
- right: comparison between results by WBM and 4th-order FDM, and analytic solutions

⟹ The WBM generates accurate results while the FDM displays artifitially attenuated results

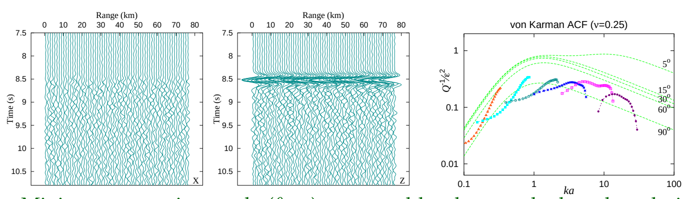
3.2 Stability test



- left: pointwise random heterogeneous medium with a standard deviation of velocity perturbation (σ) of 20 % (max. perturbation: 98 %)
- right: Von Karman random heterogeneous medium with ν=0.25, a=100 m, and σ=52% (max. perturbation: 92 %)

⟹ The WBM allows high stability even in extremely perturbed media

3.3 Implication in measured scattering attenuation rates

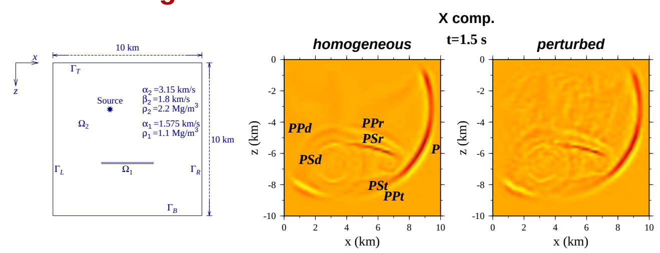


- Minimum scattering angle ($\theta_{\min}$) measured by the wavelet-based study in media perturbed by 10 %: 60-90 %

cf. A. Numerical studies based on FDM
  i) in moderately perturbed media (about 10 %) - Frankel & Clayton (1986): $\theta_{\min}$=30-45°, Roth & Korn (1993): 20-40°, Frenje & Juhlin (2000): 10-20°
  ii) in mildly perturbed media (4 %) - Jannaud *et al.* (1991): 90°
B. Analytic study - Kawahara (2002): 65°

⟹ Results by FDMs in moderately perturbed media exhibit high attenuation compared to those by the WBM, which agree with those estimated in mildly perturbed media and the analytic results. That is, the WBM can generate correct time responses while FDMs may have artificially attenuated ones.

4 Modelling in a medium with a fluid-filled crack



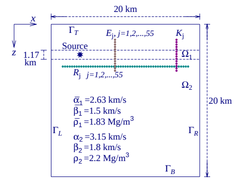
- left: a model with a fluid-filled crack and an explosive source
- middle: wave propagation in a homogeneous medium
- right: wave propagation in a randomly perturbed medium

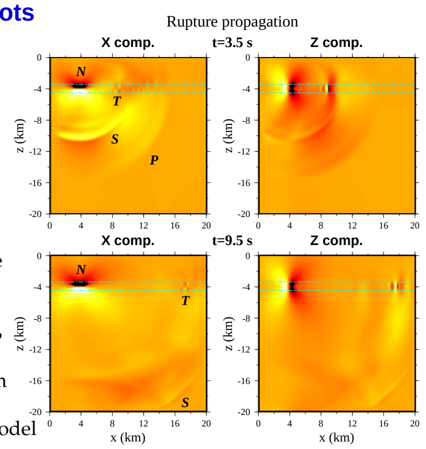
⟹ The WBM can deal localized zones of strong contrast in the scheme without introduction of additonal boundary conditions.

♠ *WBM can be extended to topography problems [2] using a grid mapping technique.*

5 Modelling in a fault zone (rupture propagation)

5.1 Model and snapshots



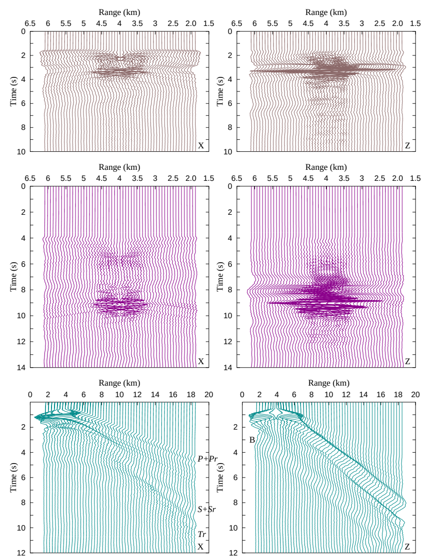


- Source: 90° dip-slip rupture with horizontal propagation
- Falut zone (Ω_1): randomly perturbed (10% for $\bar{\alpha}_1$, $\bar{\beta}_1$; 8% for $\bar{\rho}_1$) low-velocity medium
- Rupturing distance: 0.86 km
- Rupture velocity: 0.9 $\bar{\beta}_1$
- Disp. time fuction: ramp model

⟹ Permanent displacements (*N*) are found in a four-lobed pattern around the rupture plane, and considerable energy (*T*) is trapped and propagate along the fault zone.

♠ *Each fault segment can readily incorporate its own displacement time history.*

5.2 Time responses

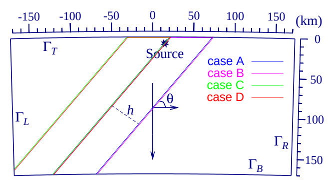


- top: (at receivers E_j), direct *P* and *S* waves are shown clearly on x components while trapped waves are dominant on z components. Signicant scattered waves are also displayed.
- middle: (at receivers K_j), various head waves arrive before direct phases inside the fault zone (x), and the rupture propagation makes uneven energy distribution inside the fault zone (V-shape travel time in x) and discontinuity in wave trains (z).
- bottom: (at receivers R_j), various multi-reflected phases are following direct phases and trapped waves continuously smear into the background medium as scattered waves.

⟹ The WBM can consider realistic dislocation sources with complex forms (i.e., undifferentiable in time) of displacement time function in heterogeneous media in a direct way.

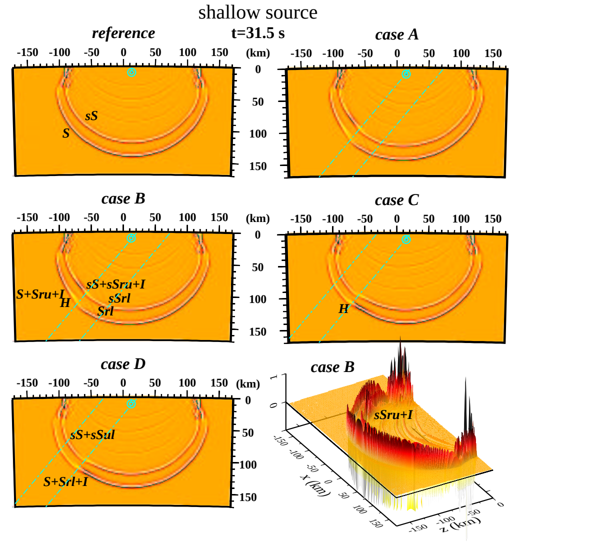
6 Modelling in subduction zones (*SH* waves)

6.1 Simplified slab models



The source lies
- case A: just below the upper boundary
- case B: just above the upper boundary
- case C: just below the lower boundary
- case D: just above the lower boundary
- background: *ak135* (Kennett *et al.*, 1995)
- slabs: 5 % velocity anomaly, h=40 km, θ=50°

6.2 Snapshots



⟹ The relative source position plays an important role in wavefront variation. Significant interface waves and critically reflected waves are observed, and they affect wave trains recorded in a free surface. In tectonic regions, hypocenters are naturally close to coherent tectonic structures and initiated waves are affected severely even by small-contrast structures.

7 Conclusions

For realistic modelling in tectonic regions, where hypocenters are close to (or inside) tectonic structures, numerical techniques which can deal with dynamic source process and heterogeneities in source region accurately are needed. The wavelet-based method (WBM) provides a good representation for a wide range of such studies. The strong points of the WBM are:

- The WBM has high accuracy and stability, which allow accurate modelling in highly heterogeneous media, and in complex media with strong contrasts. The WBM provides correct time responses for quantitative seismic studies.
- The WBM can implement complex dynamic sources and can readily consider different displacement time history at each segment of rupture plane.
- These sources can be implemented directly in heterogeneous source region with no need of homogeneity in source regions.

References

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