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A Wavelet-Based Method for Simulation of Seismic Wave Propagation

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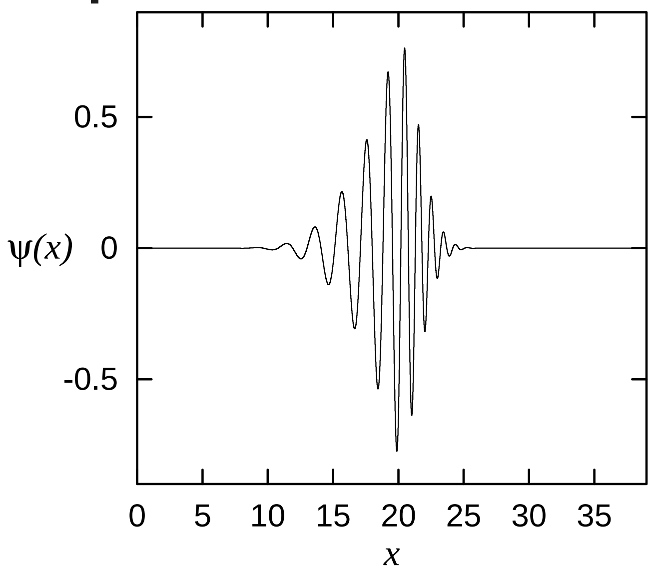
Abstract

- **Seismic wave propagation (e.g., both *P-SV* and *SH* in 2-D) can be modeled using wavelets. The governing elastic wave equations are transformed to a first-order differential equation system in time with a displacement-velocity formulation. Spatial derivatives are represented with a wavelet expansion using a semigroup approach. The evolution equations in time are derived from a Taylor expansion in terms of wavelet operators. The wavelet representation allows high accuracy for the spatial derivatives. Absorbing boundary conditions are implemented by including attenuation terms in the formulation of the equations. The traction-free condition at a free surface can be introduced with an equivalent force system. Irregular boundaries can be handled through a remapping of the coordinate system. The method is based on a displacement-velocity scheme which reduces memory requirements by about 30% compared to the use of velocity-stress. The new approach gives excellent agreement with analytic results for simple models including the Rayleigh waves at a free surface. A major strength of the wavelet approach is that the formulation can be employed for highly heterogeneous media and so can be used for complex situations.**

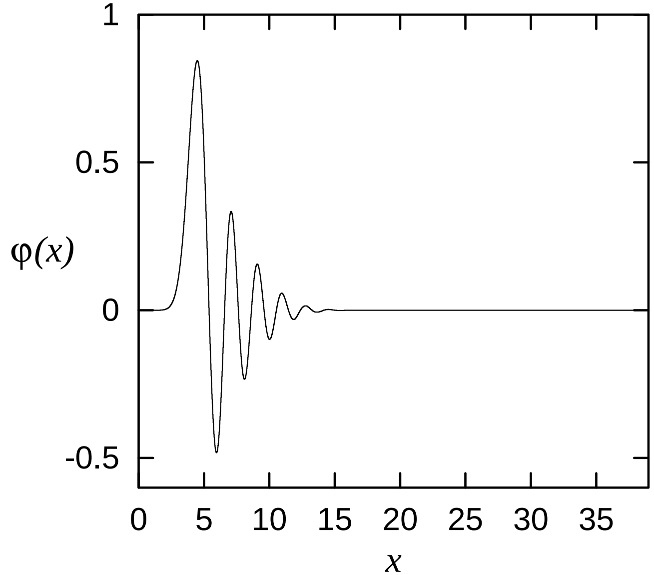
Wavelets & Applications (I-1)

- **Signal processing**
  - Phase picking (*P*, *S*, *Lg*)
  - Estimation of a time-varying spectral density matrix for three component seismic data
  - Measurement of the anisotrophy at a given area using a relative phase between components
- **Numerical analysis**
  - Simplification of governing equations by applying a wavelet function
  - Composite use of a FD scheme and an interpolating wavelet transform
  - Application of wavelets in differentiation of functions

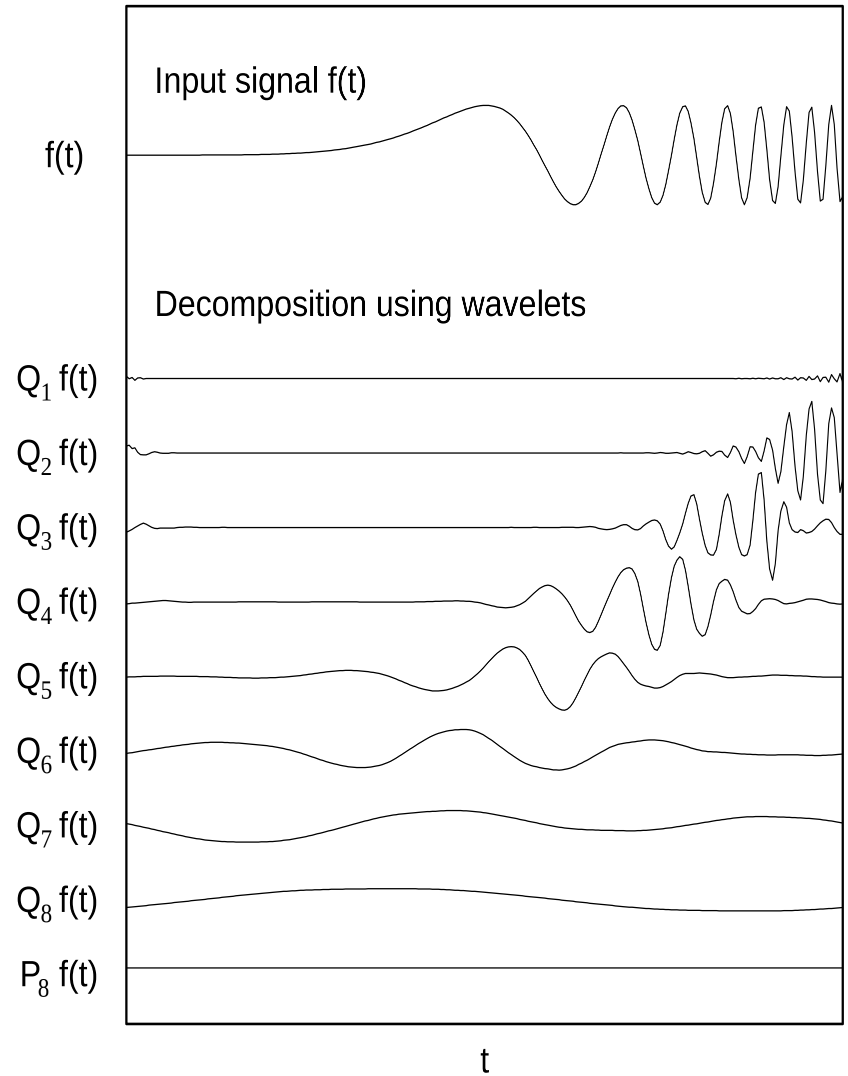
Wavelets & Applications (I-2)



Wavelet and scaling functions



Example of signal decomposition



Wavelets & Applications (I-3)

- **Discrete wavelet transform (DWT)**
  - Daubechies (1992) built the foundation of a DWT
  - Based on a multiresolution analysis that decomposes a signal into components of different scales
- **Wavelets & parabolic PDEs**
  - Beylkin & Keiser (1997) introduced a numerical scheme for parabolic PDEs using semigroup approach and wavelets
  - Semigroup approach

$$\partial_t g = \mathcal{L} g + \mathcal{N} f(g)$$

$$\Rightarrow g_{n+1} = e^{\delta t \mathcal{L}} g_n + \delta t \left(\gamma N_{n+1} + \sum_{m=0}^{R-1} \beta_m N_{n-m} \right)$$

A Wavelet-Based Method (II-1)

- Spatial derivative operators in elastic wave equations are treated through wavelet transforms in a physical domain
- The resulting second order differential equations for time evolution are solved via a system of first-order differential equations using a displacement-velocity formulation, which can reduce memory requirements by 30 % compared to a velocity-stress formulation
- With the combined aid of a semigroup representation and spatial differentiation using wavelets, a uniform numerical accuracy of spatial differentiation can be maintained across the domain

A Wavelet-Based Method (II-2)

- Numerical difficulties
  - Numerical formulation of wave equations for the parabolic PDE scheme
  - Representation of matrix operators in wavelet bases
  - Treatment of artificial boundary conditions
  - Treatment of inherent periodic boundary conditions
  - Implementation of external boundary conditions (e.g. free-surface conditions, rigid boundary conditions)
  - Expansion to complex structure problems

A Wavelet-Based Method (II-3)

- Corresponding remedies
  - Introduction of a displacement-velocity formulation
  - Application of a Taylor expansion for the simplification of matrix operators
  - Introduction of absorbing boundary conditions
  - Adjustment of physical parameters around the artificial boundaries
  - Application of equivalent force terms in the system of equations for external boundary conditions
  - Introduction of grid generation scheme

Formulation of Equations (III)

- **2-D elastic wave equations (*P-SV*)**

$$\left.\begin{aligned}\frac{\partial^2 u_x}{\partial t^2} &= \frac{1}{\rho}\left(\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xz}}{\partial z}+f_x\right)\\ \frac{\partial^2 u_z}{\partial t^2} &= \frac{1}{\rho}\left(\frac{\partial\sigma_{xz}}{\partial x}+\frac{\partial\sigma_{zxz}}{\partial z}+f_z\right)\end{aligned}\right\}\Rightarrow \partial_t\mathbf{U}=\mathbf{LU}+\mathbf{F}$$

where,

$$\mathbf{U}=\begin{pmatrix}u_x\\ v_x\\ u_z\\ v_z\end{pmatrix}\quad \mathbf{L}=\begin{pmatrix}0 & I & 0 & 0\\ L_{xx} & 0 & L_{xz} & 0\\ 0 & 0 & 0 & I\\ L_{zx} & 0 & L_{zz} & 0\end{pmatrix}\quad \mathbf{F}=\frac{1}{\rho}\begin{pmatrix}0\\ f_x\\ 0\\ f_z\end{pmatrix}$$

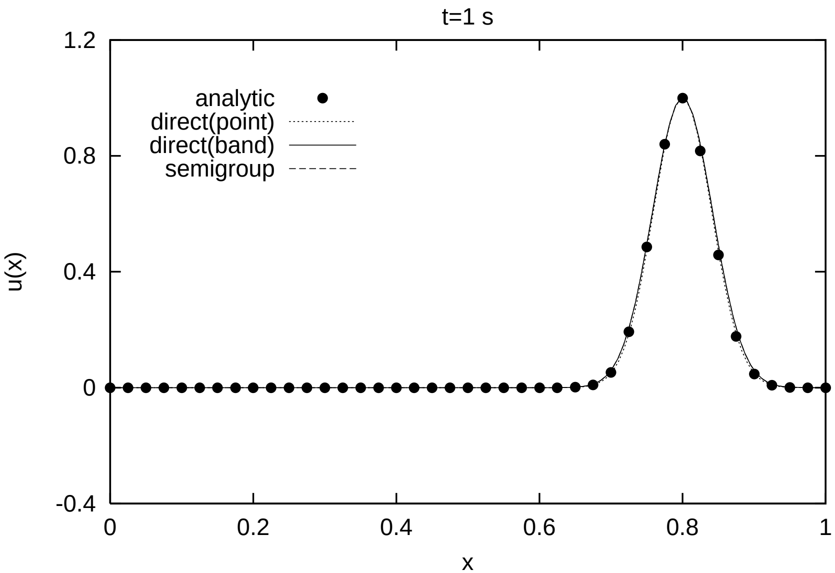
U : vector variable

L : matrix operator

F : force vector

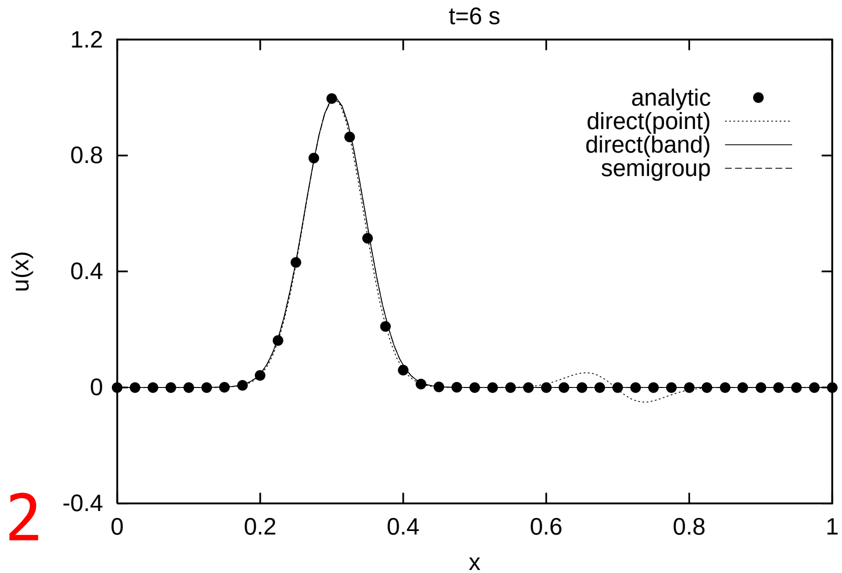
Test of Equivalent Forces (IV)

- **1-D acoustic waves**



1

2



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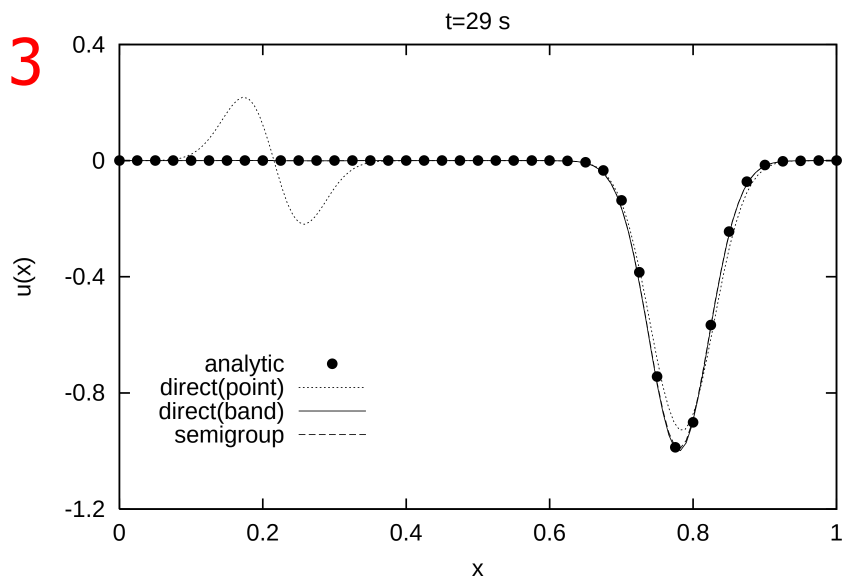
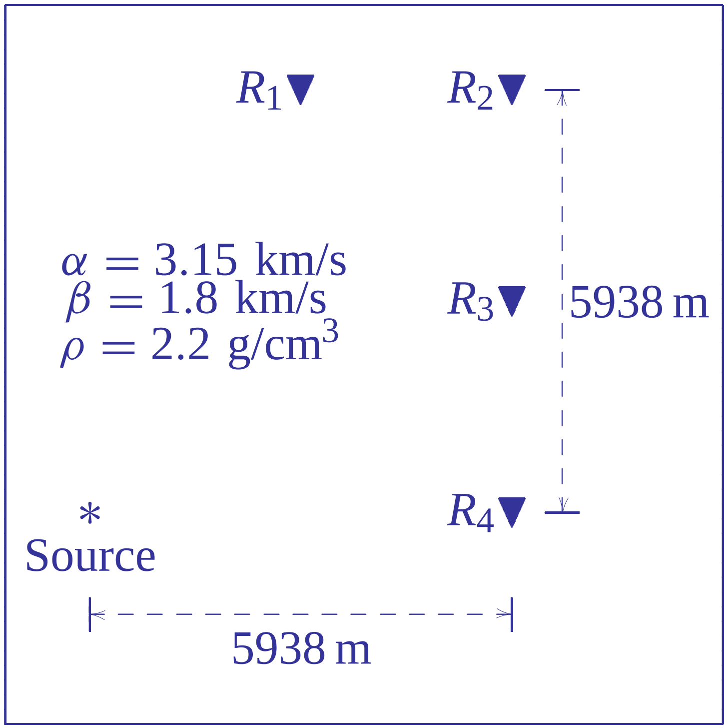


The equivalent force terms for boundary conditions can simulate exactly the effects which are caused by the presence of the boundaries

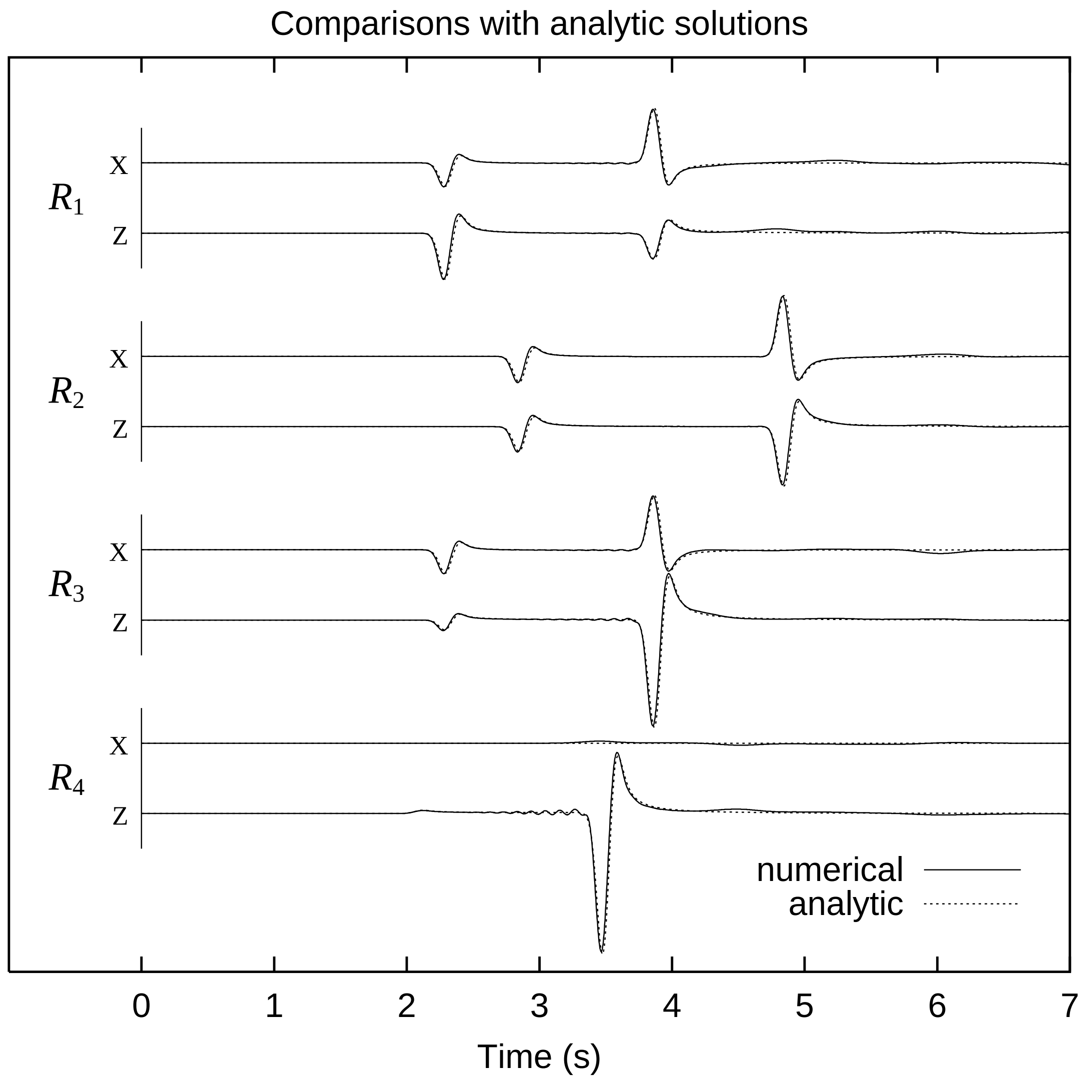
Unbounded Medium (V)

- ***P-SV* waves**



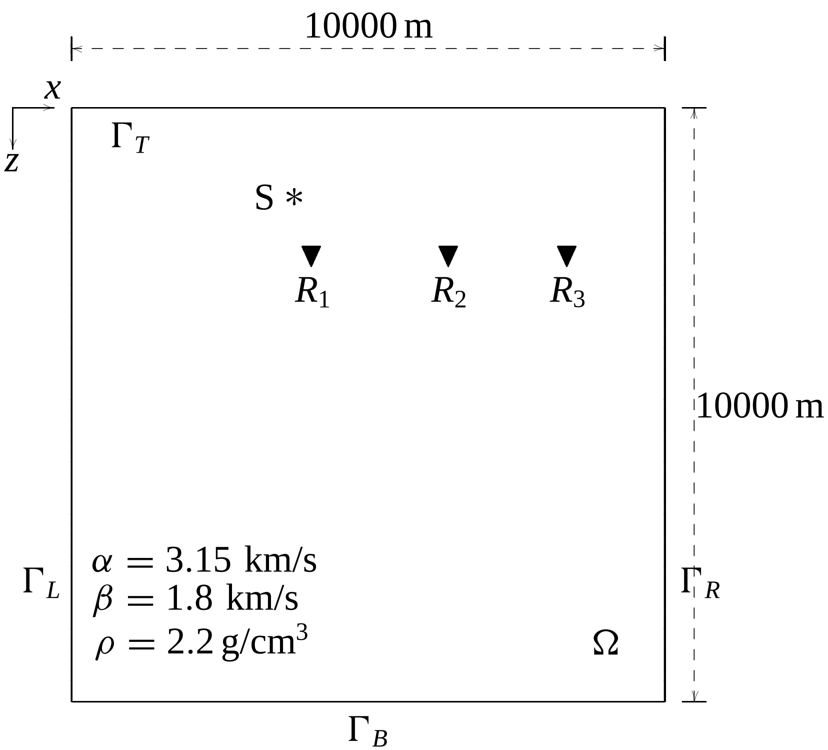
Model

Comparisons with analytic solutions

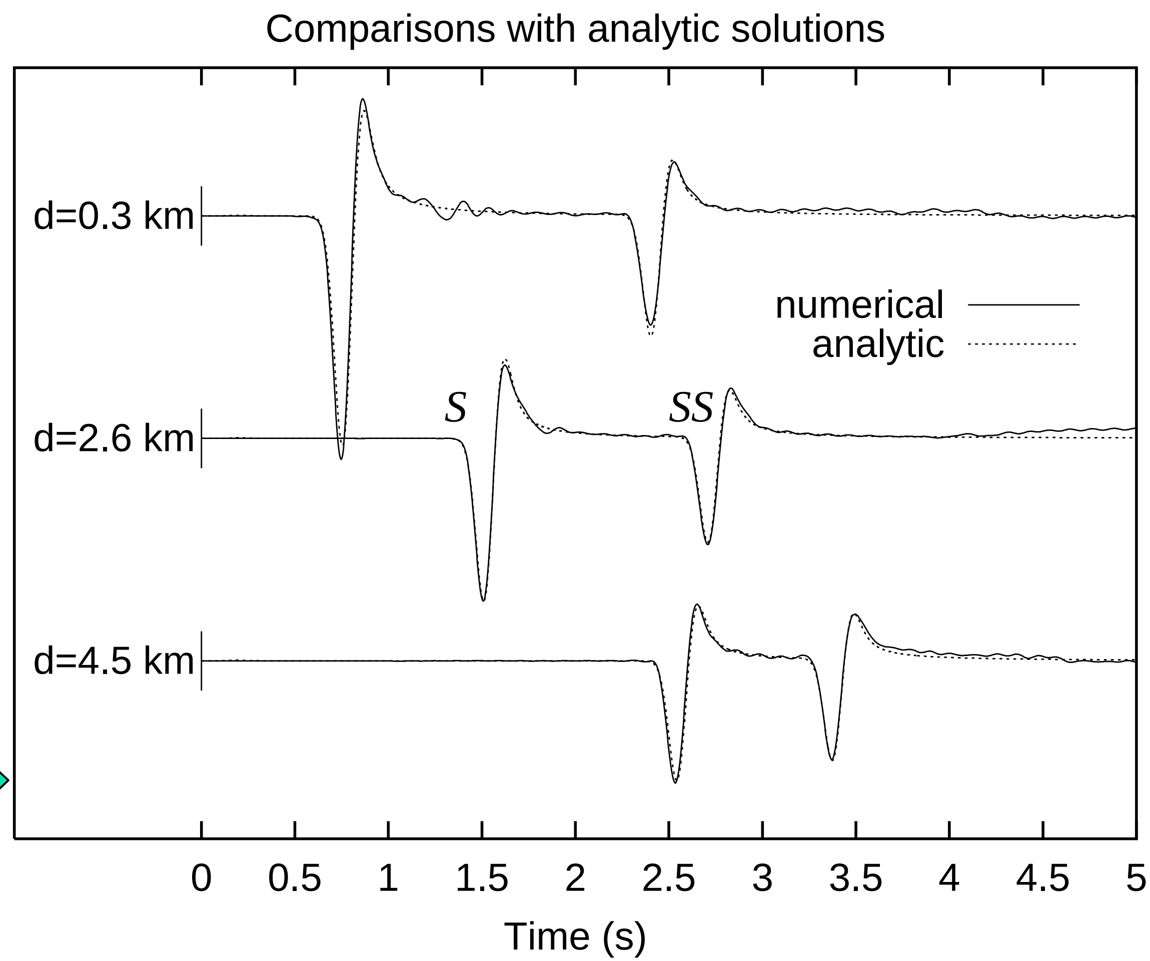


Media with a Free Surface (VI-1)

- ***SH* waves**



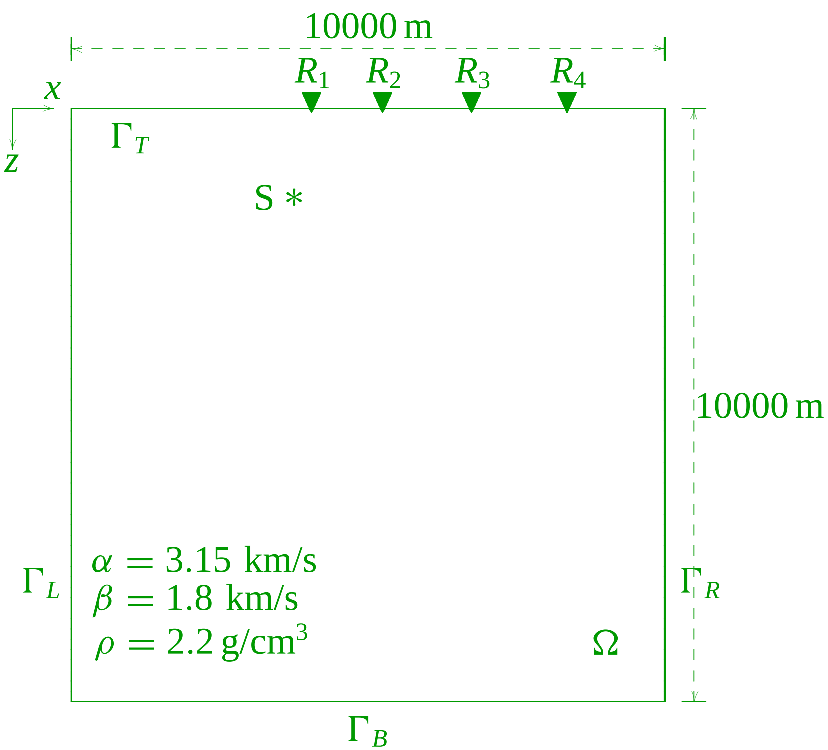
Model



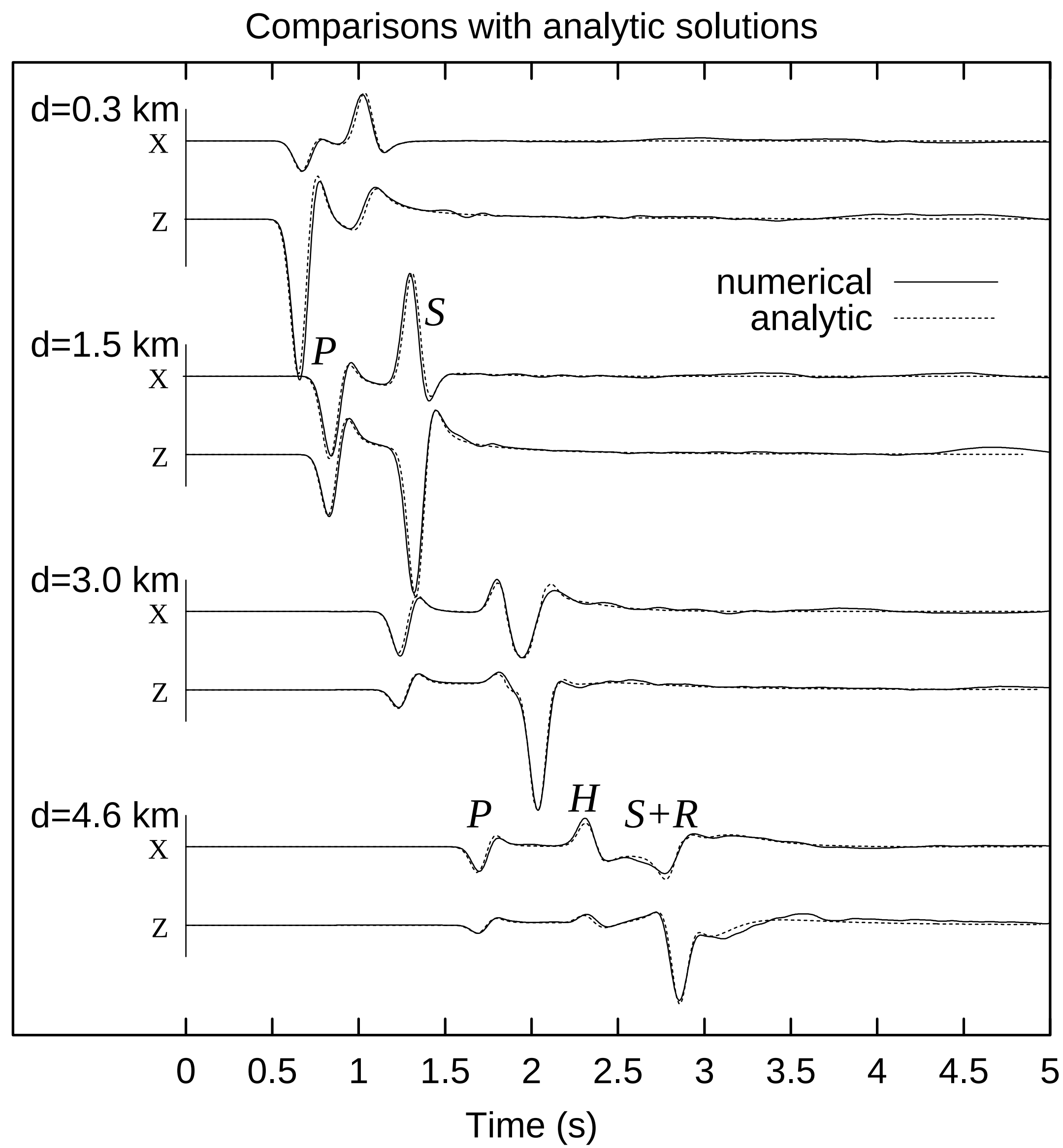
Comparisons with analytic solutions

Media with a Free Surface (VI-2)

- ***P-SV* waves**



Model



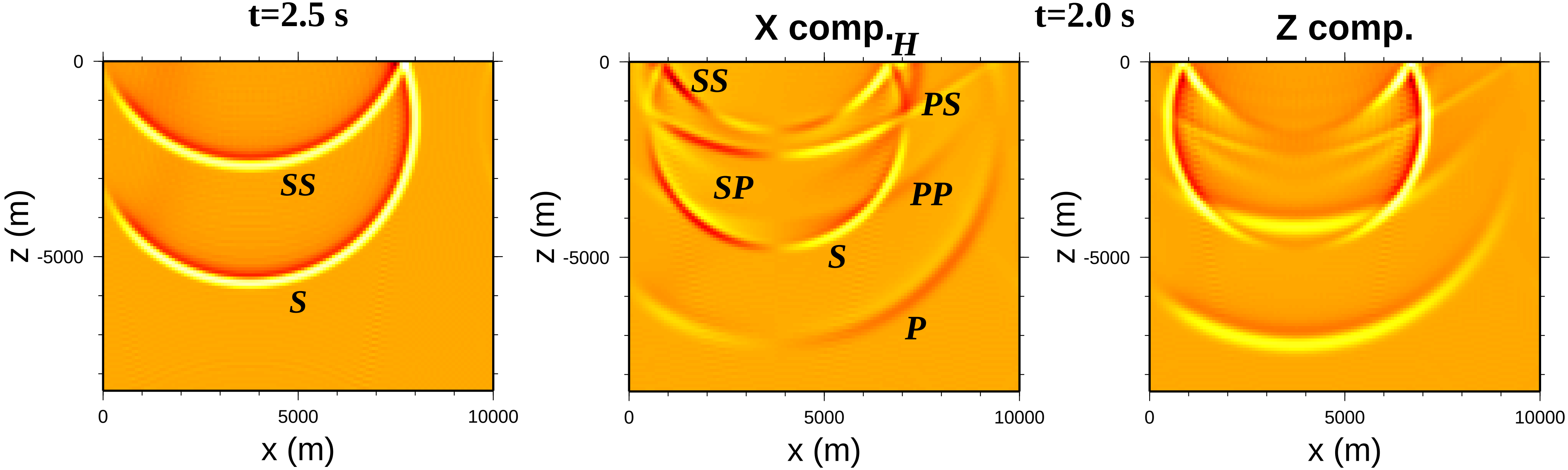
Comparisons with analytic solutions

Media with a Free Surface (VI-3)

- Snapshots of wavefields

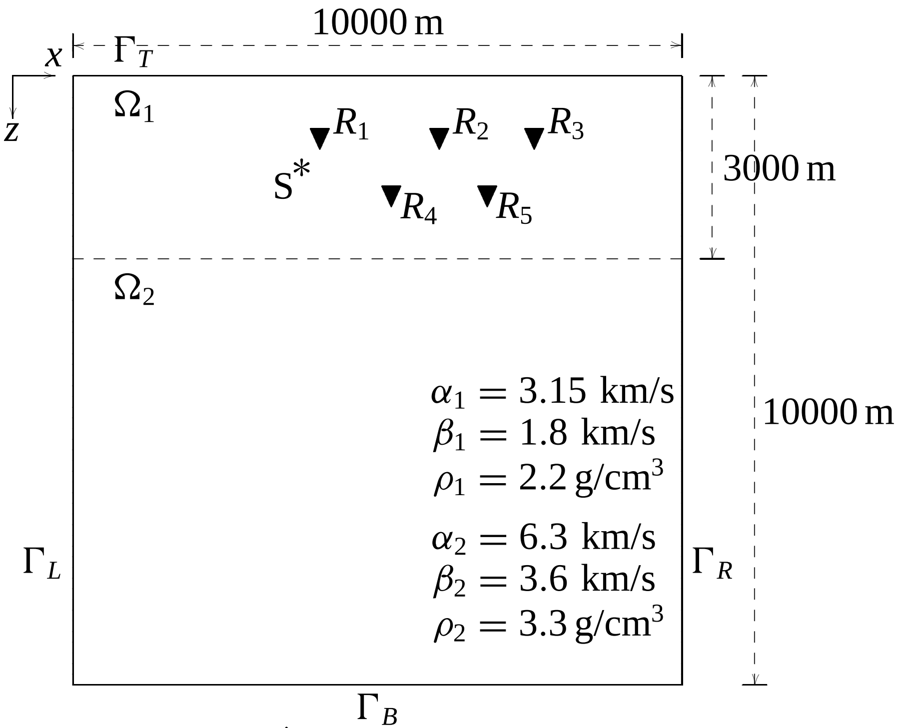
SH waves t=2.5 s

P-SV waves t=2.0 s

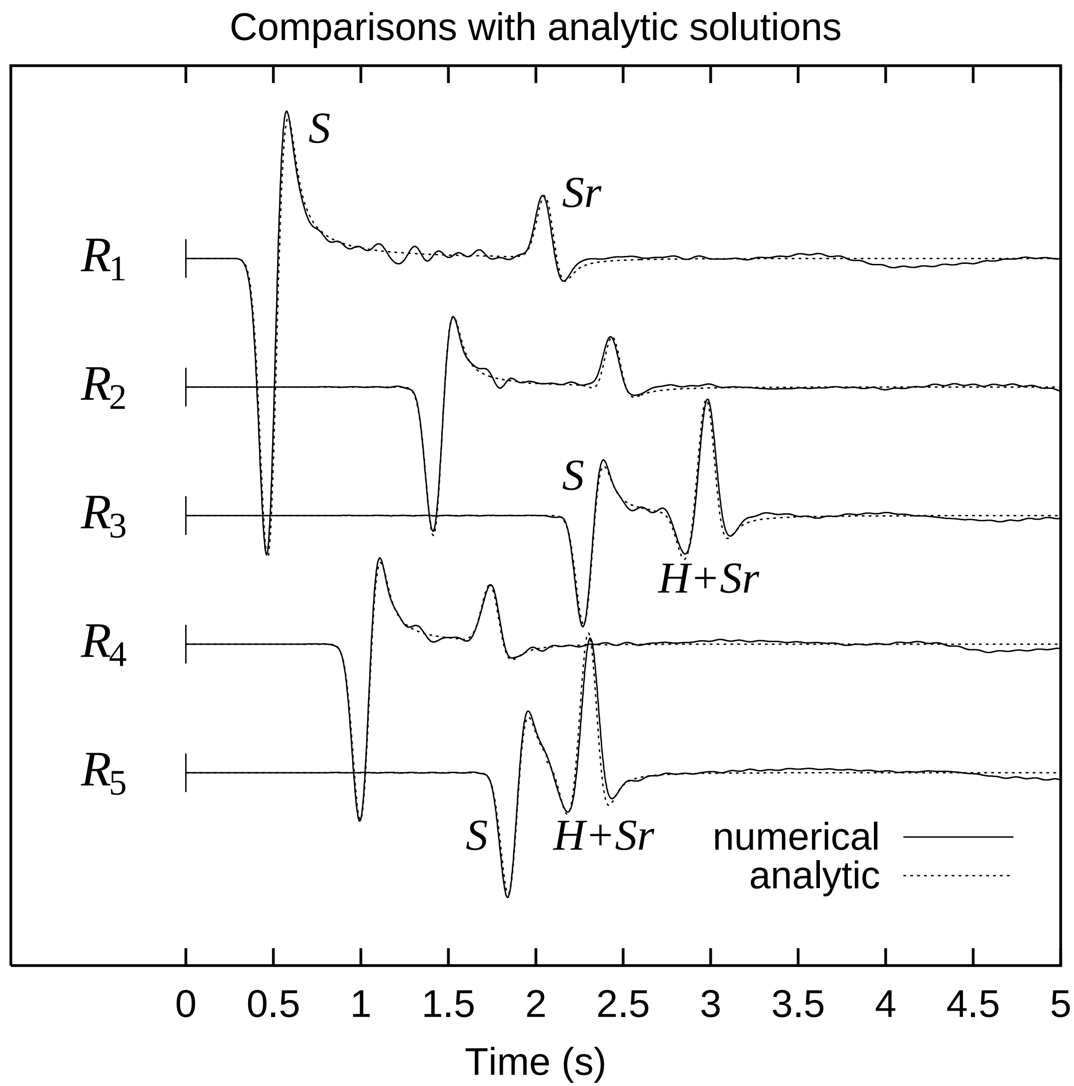


Two-Layered Media (VII-1)

- ***SH* waves**



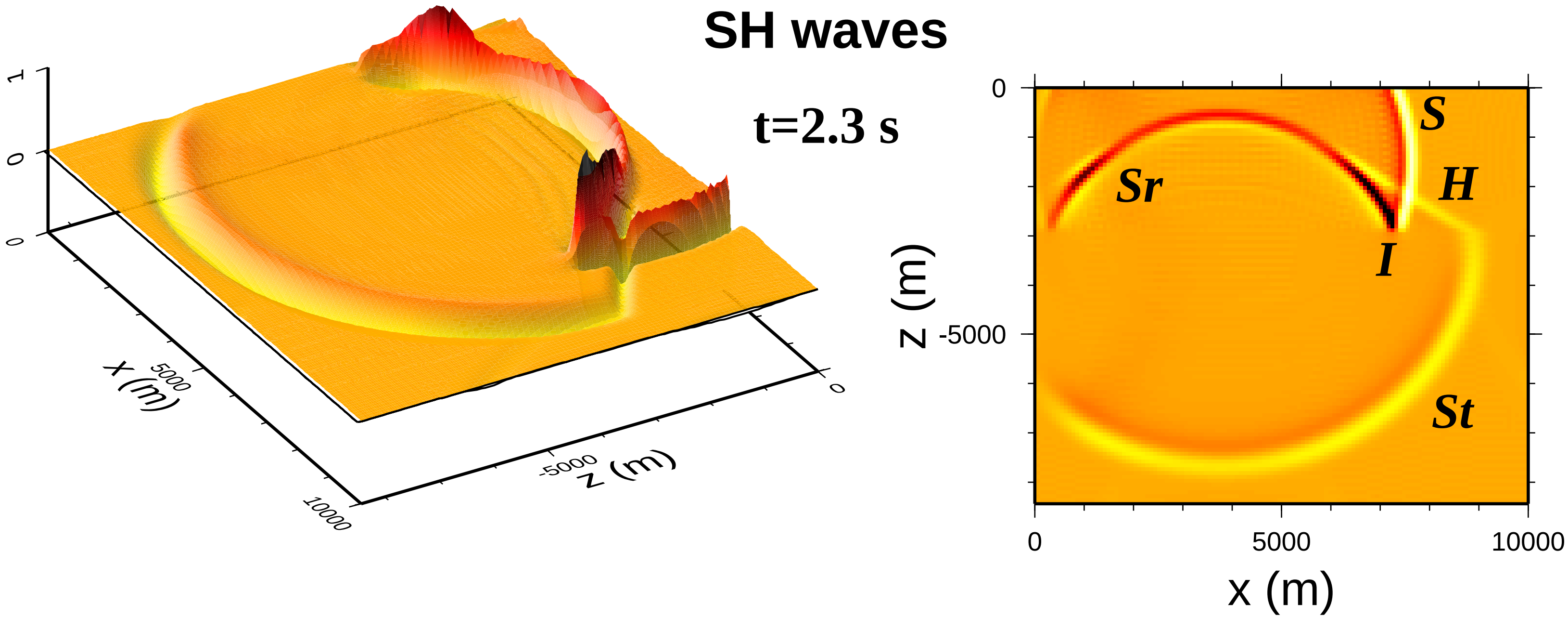
Model



Comparisons with analytic solutions

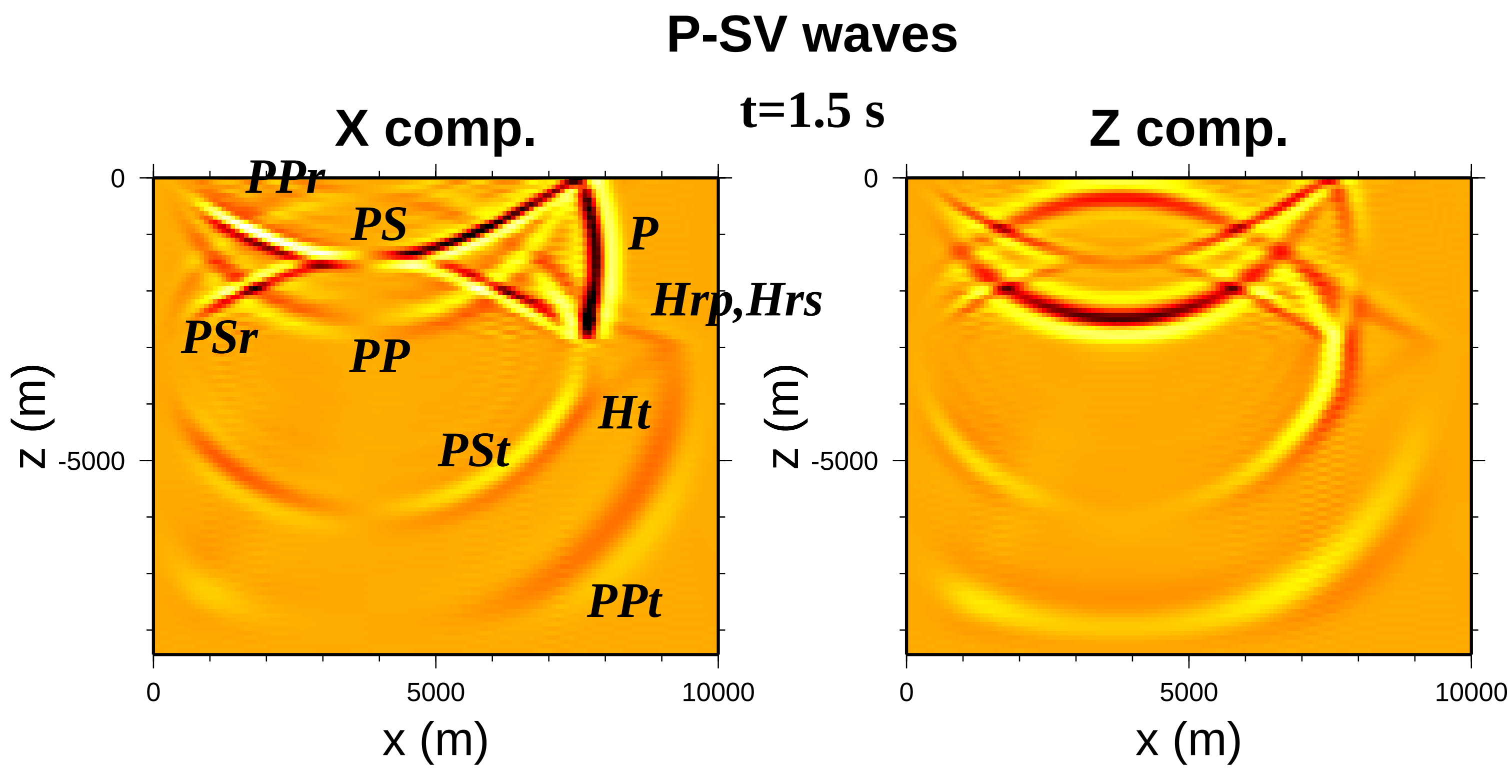
Two-Layered Media (VII-2)

- **Snapshots of *SH* wavefields**



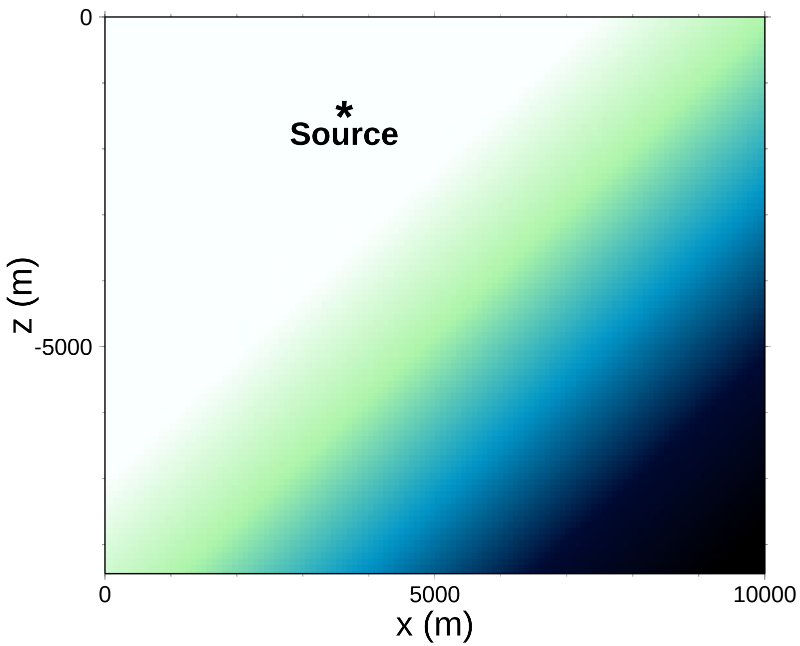
Two-Layered Media (VII-3)

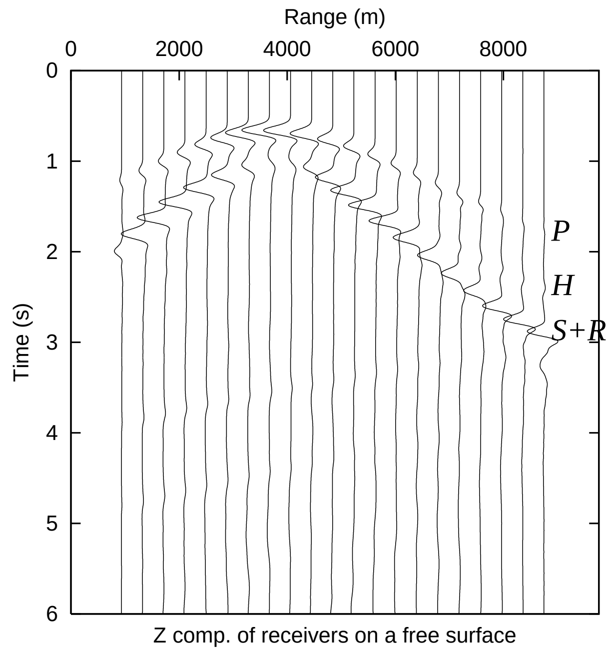
- Snapshots of *P-SV* wavefields

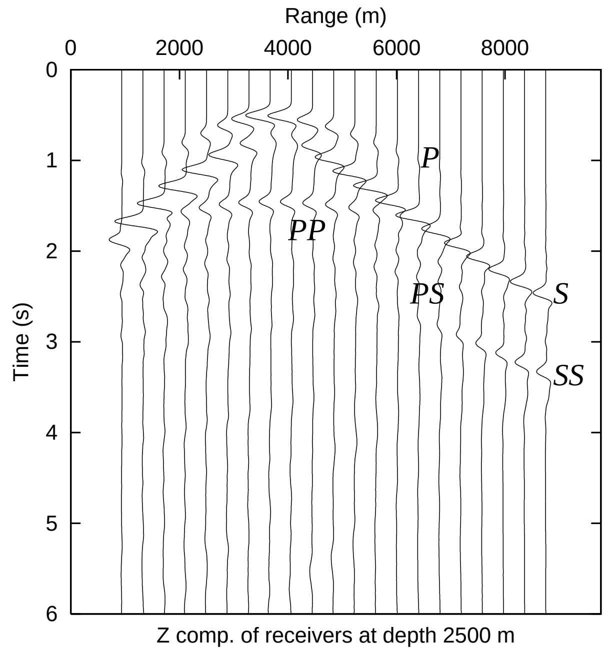


Gradient Velocity Media (VIII-1)

- Model & Time responses







Slant : 38.7°

ρ: 2.2~3.85 g/cm^3

α: 3.15~7.88 km/s

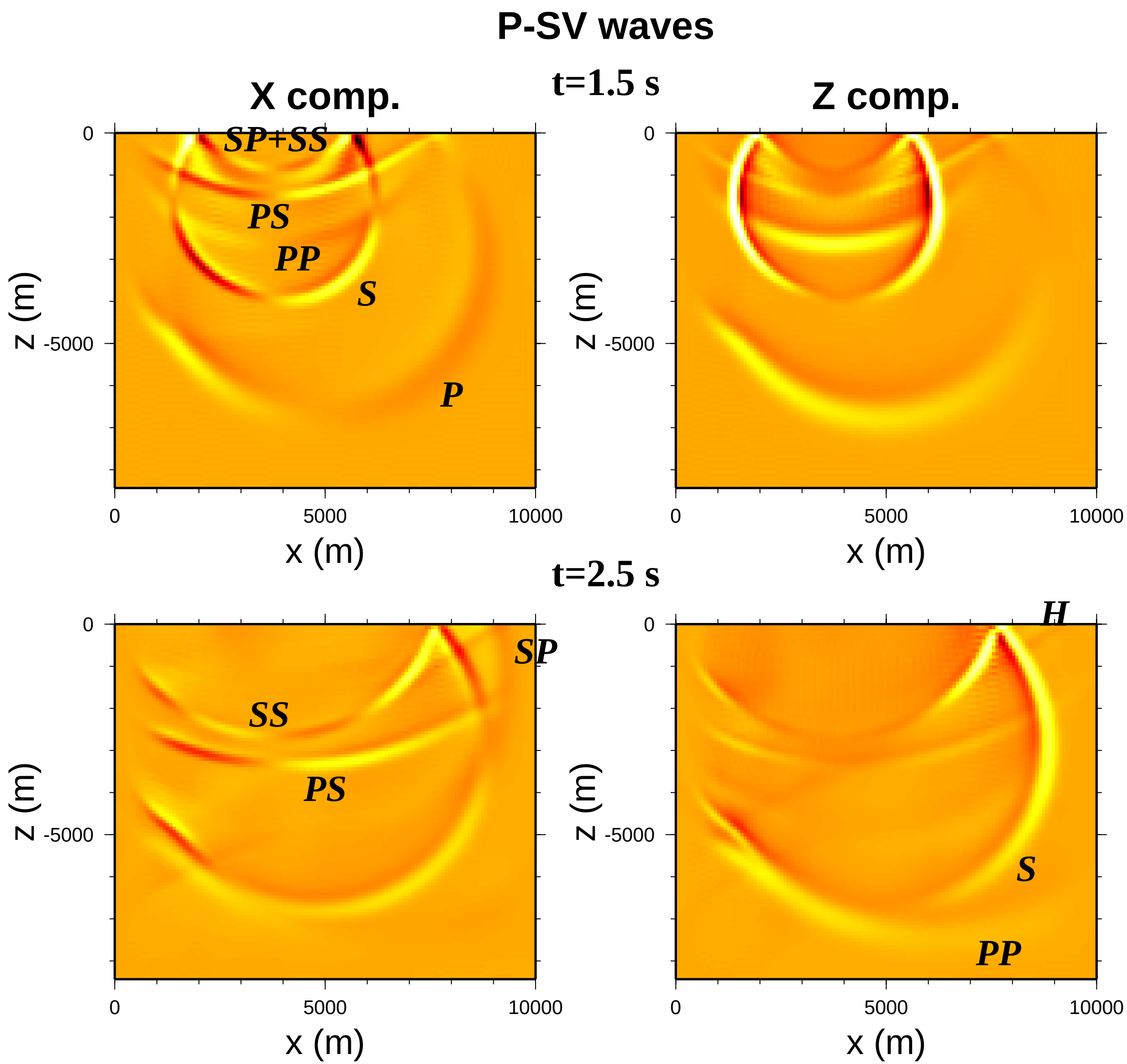
β: 1.8~4.5 km/s

Vertical components of receivers on a free surface

Vertical components of receivers at depth 2500 m

Gradient Velocity Media (VIII-2)

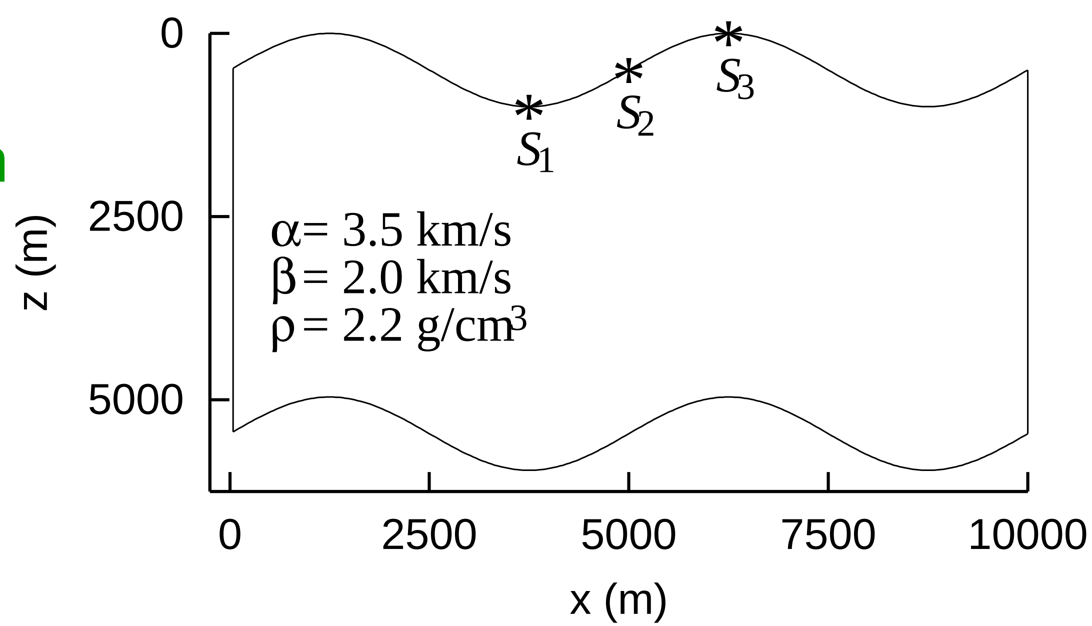
- Snapshots of *P-SV* wavefields



Media with Topographic Surface (IX-1)

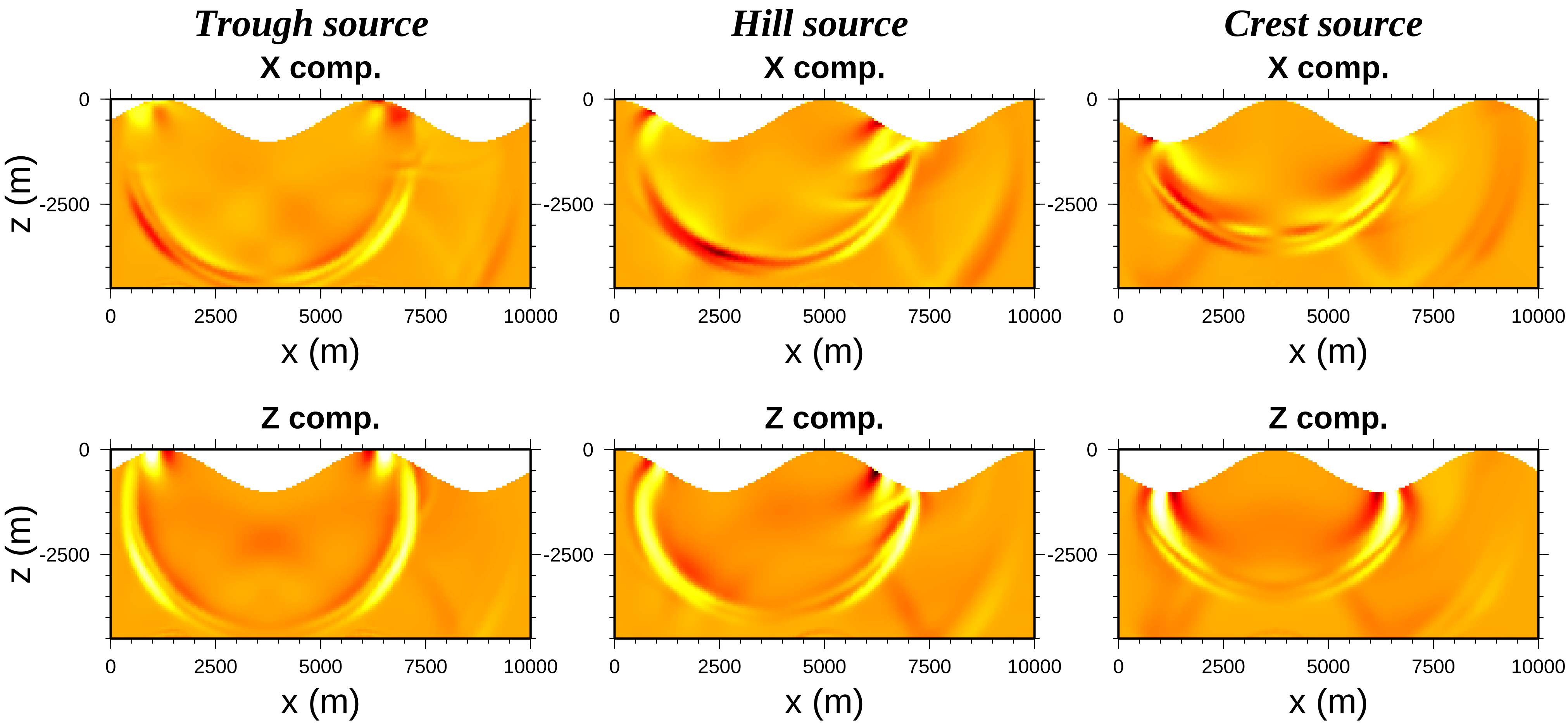
- Application to topographic problems
  - Introduce a grid generation scheme : mapping a rectangular grid system to a curved grid system with consideration of physical topography

- Numerical model
  - A homogeneous medium with a sinusoidal free surface
  - Three different positions of sources; at a trough, a hill and a crest beneath a free surface



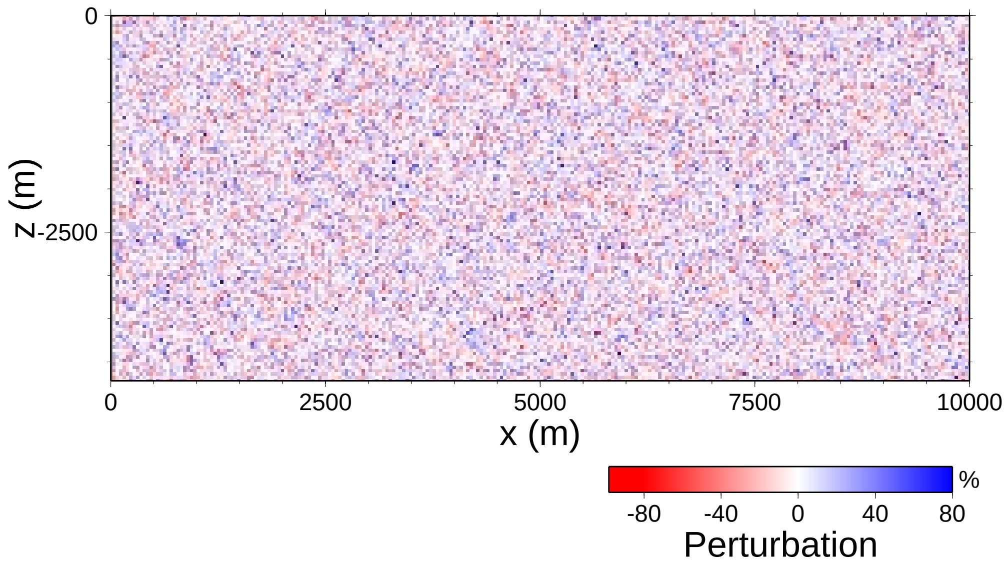
Media with Topographic Surface (IX-2)

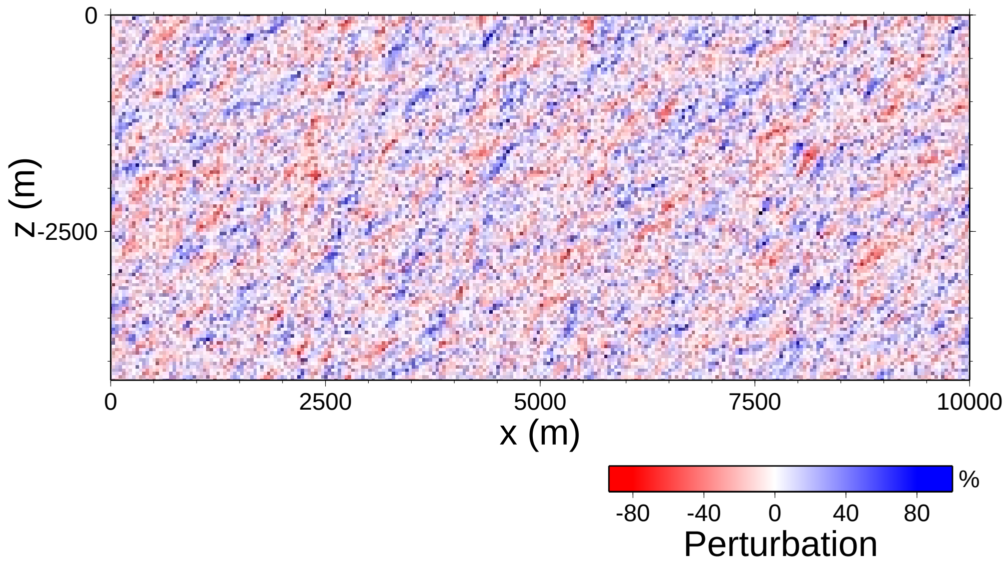
- Snapshots of *P-SV* wavefields (t=1.9 s)



Random Heterogeneous Media (X-1)

- Numerical models



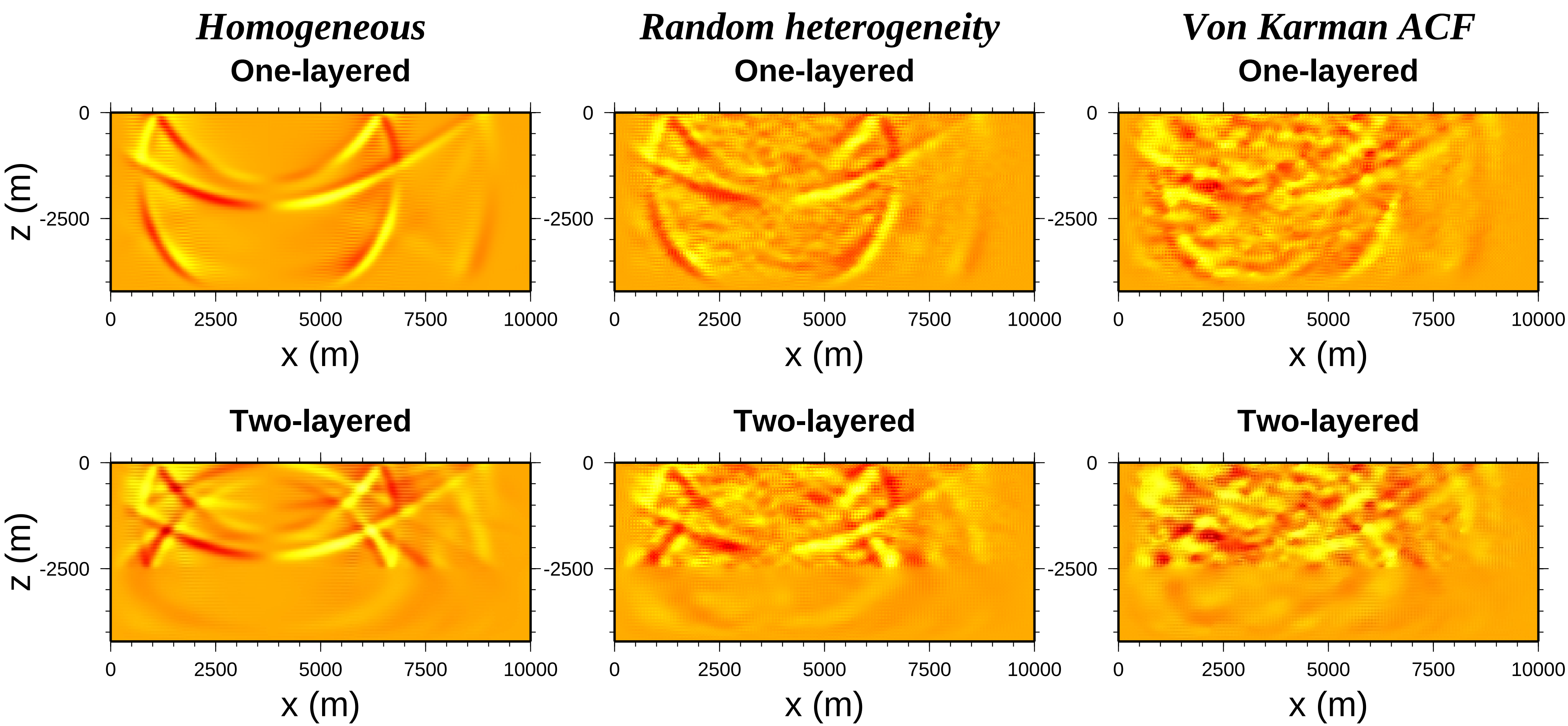


Pointwise random heterogeneous medium with Gaussian probability distribution and standard deviation of 20 % in velocity

Stochastic heterogeneous medium using a Von Karman autocorrelation function with a correlation distance of 5 km

Random Heterogeneous Media (X-2)

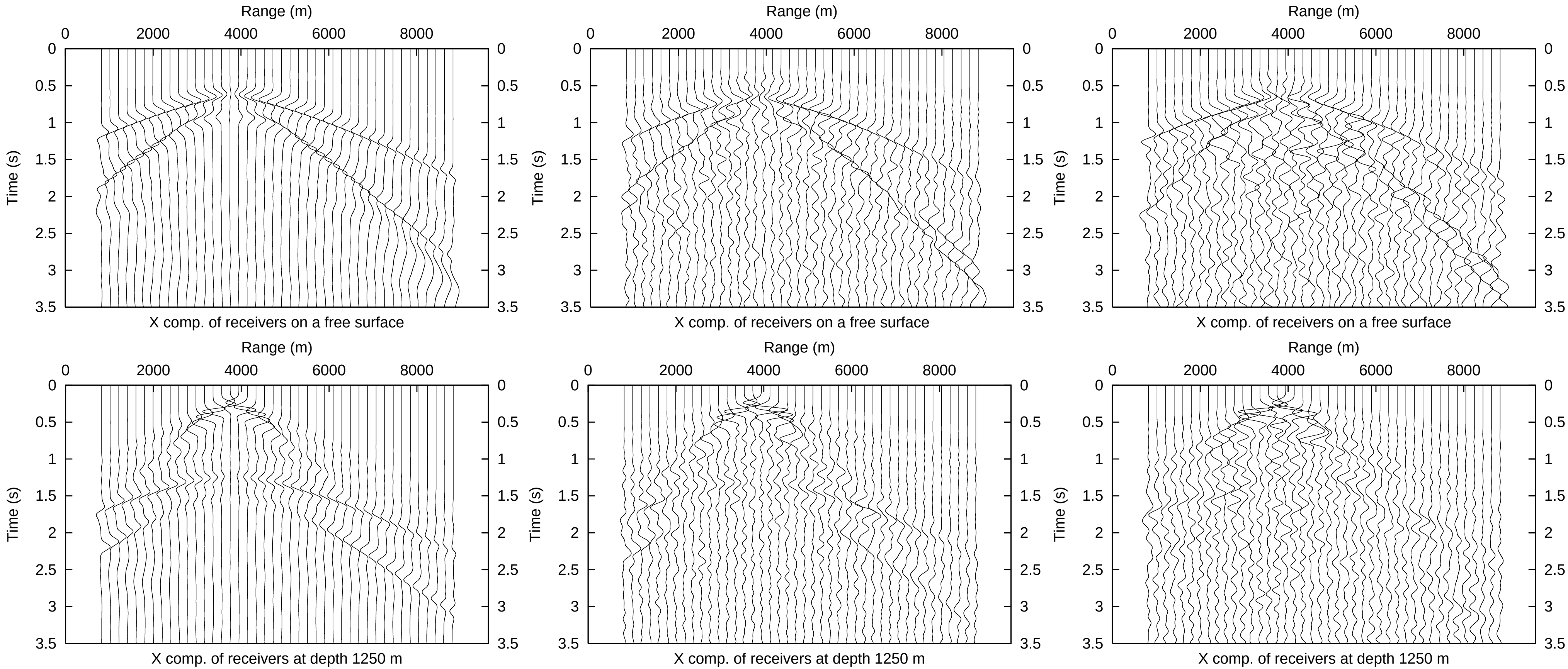
- **Snapshots of *P-SV* wavefields (t=1.9 s, *X* comp.)**



Random Heterogeneous Media (X-3)

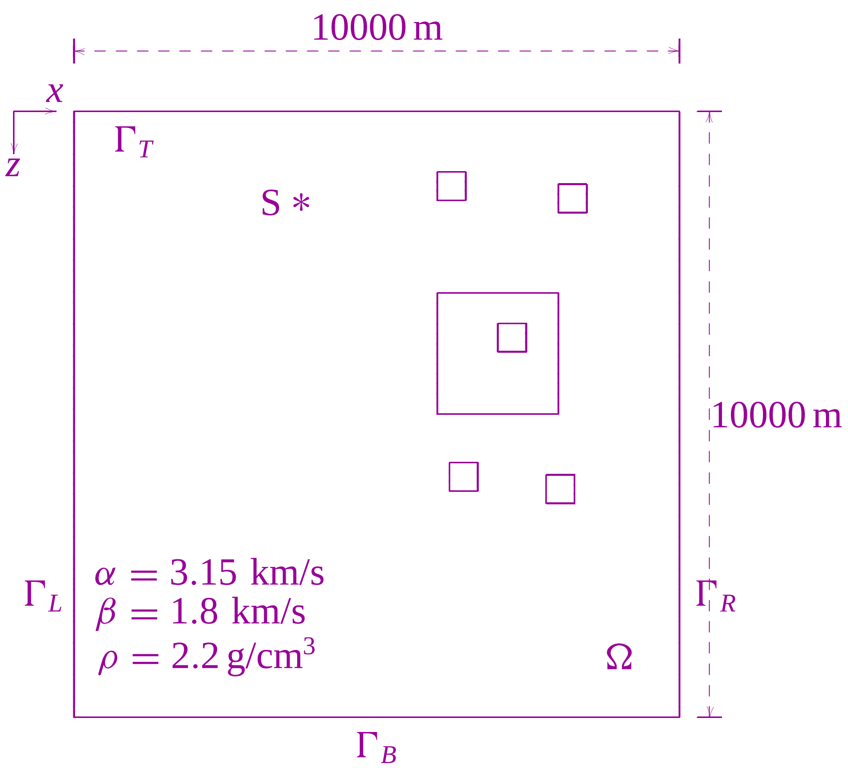
- Time responses (one-layered media, *X* comp.)

  - Homogeneous
  - Random heterog.
  - Von Karman ACF



Media with Cavities (XI-1)

- **Numerical model**



One or five cavities inside a medium

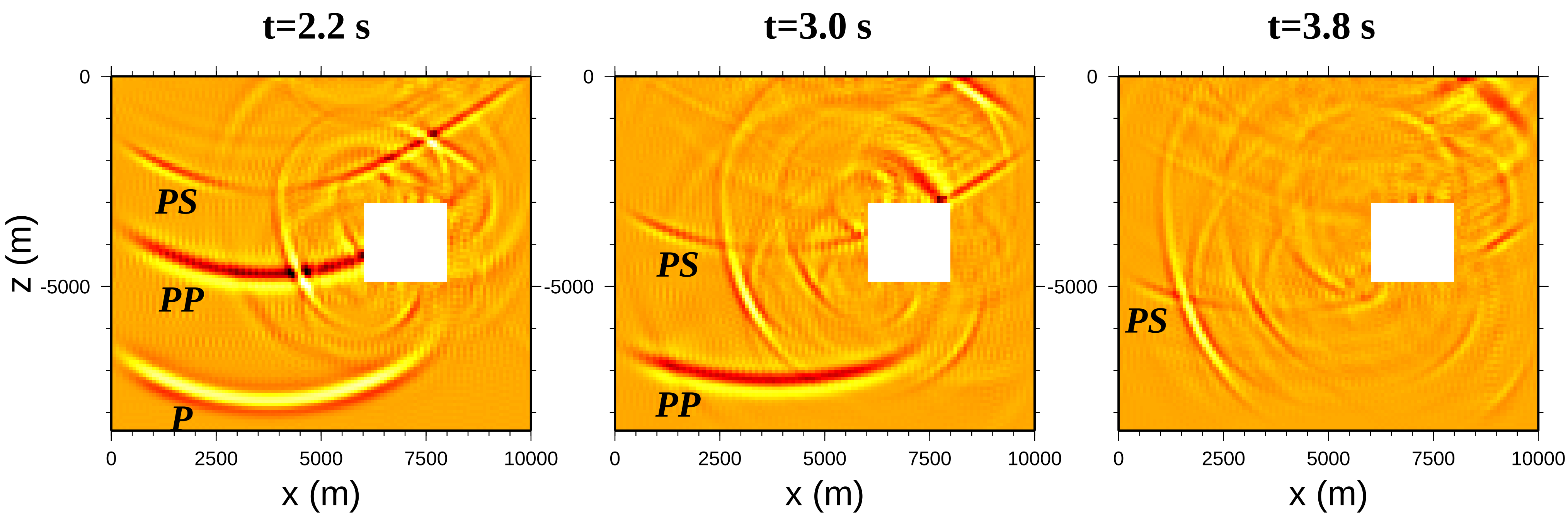
- **Implementation of boundary conditions**

-via additional equivalent force terms (i.e. traction-free boundary conditions)

-can implement various boundary conditions at various locations at the same time

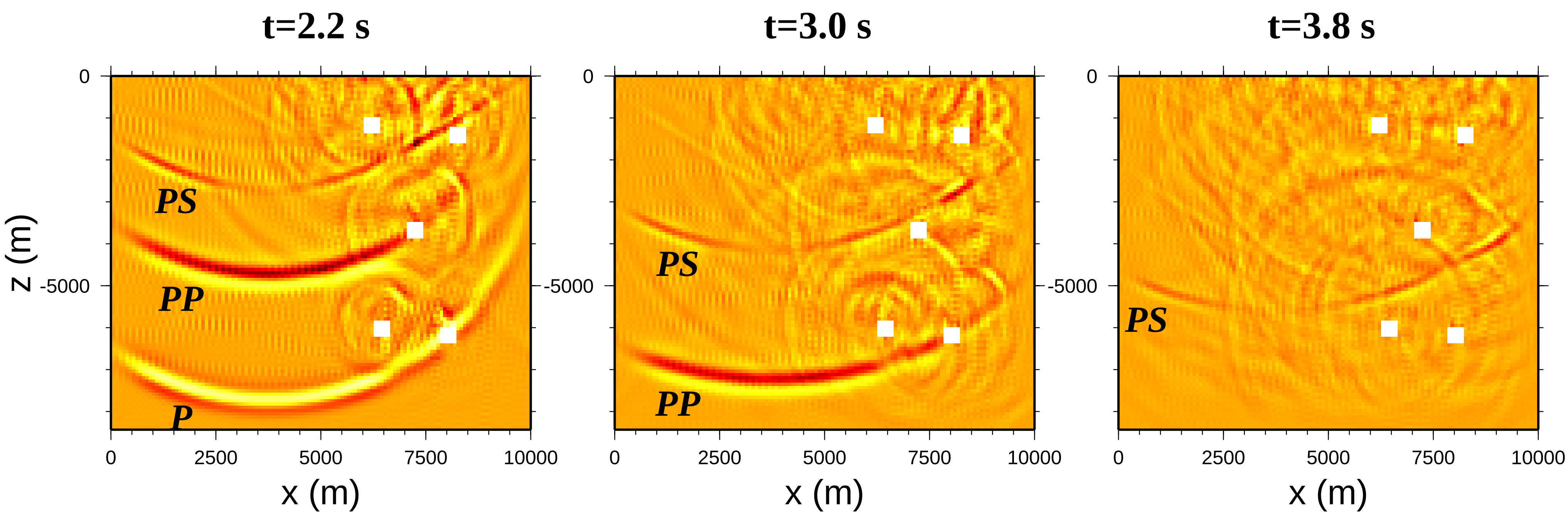
Media with Cavities (XI-2)

- Snapshots in one cavity model (*Z* comp.)



Media with Cavities (XI-3)

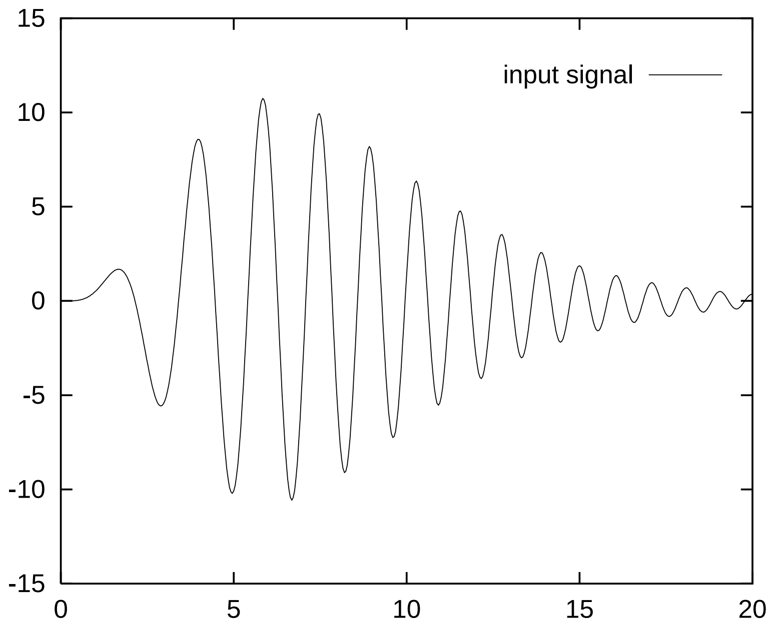
- **Snapshots in five cavities model (*Z* comp.)**



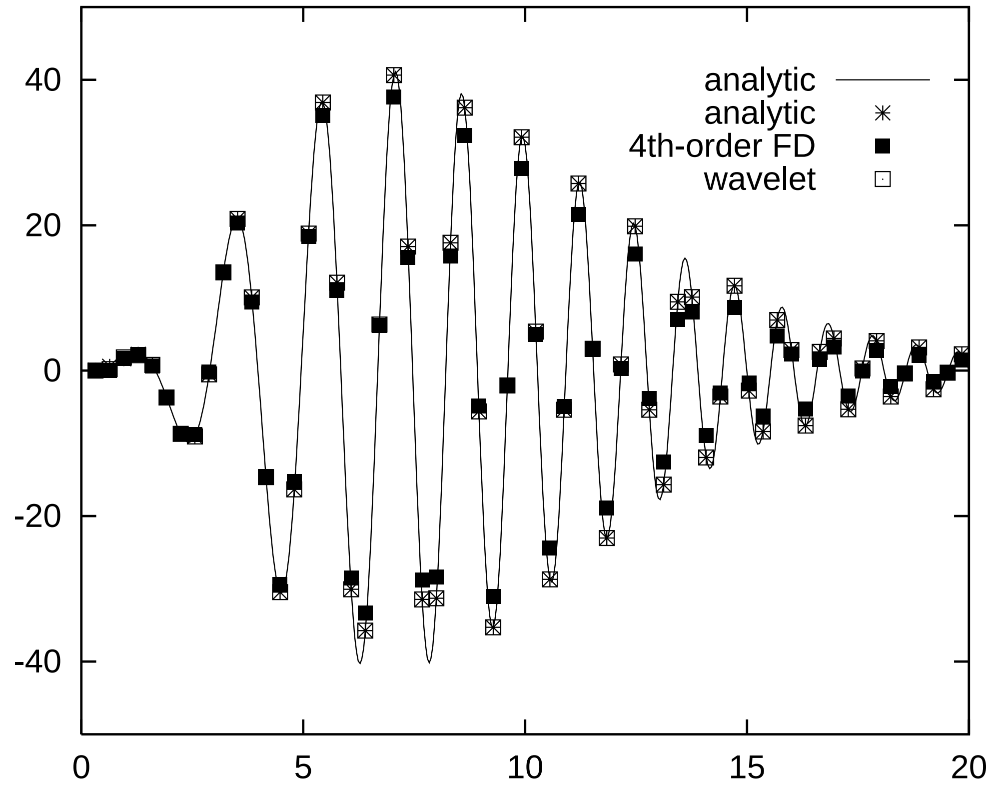
Tool for Quantitative Study (XII-1)

- **Comparison of accuracy with 4^{th}-order FDM**

➔ **Apply the first-order differentiation to the input signal which is corresponding to the quick variation of physical parameters in random heterogeneous media**

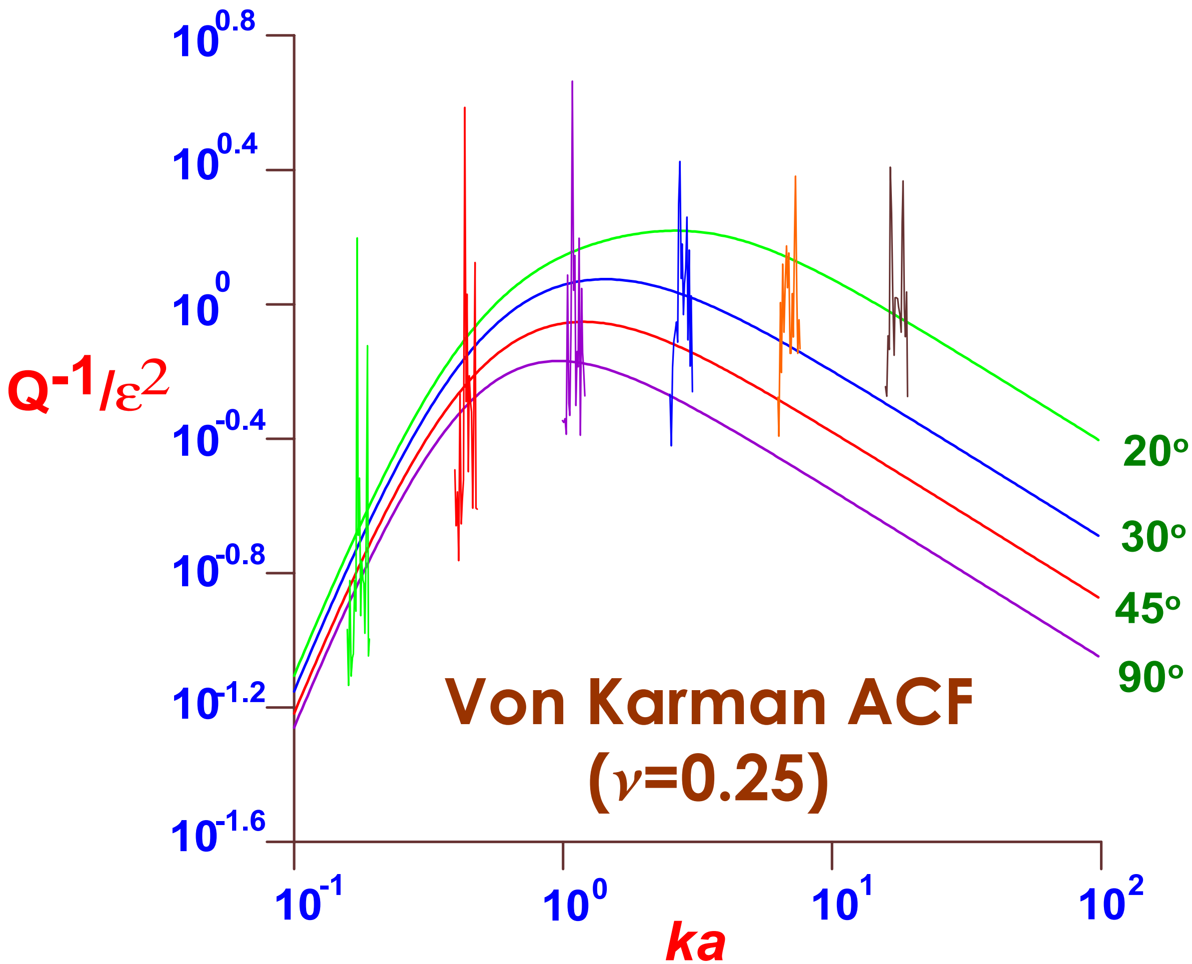


where, $N_X = 64$



Tool for Quantitative Study (XII-2)

- **Application to Q estimation**



Cf. Other Researches

-Frankel & Clayton (1986) : θmin = 30-45°

-Roth & Korn (1993) : θmin = 20-40°

-Jannaud *et al.* (1991) : θmin = 90°

Conclusions (XIII)

- Can obtain high accuracy of numerical responses of media for wave propagation due to almost no loss of accuracy during spatial differentiation and third order accuracy for time derivatives
- Stable in highly perturbed velocity media – Numerically stable scheme
- Can implement various complex boundary conditions at various locations easily using equivalent force terms
- Energy is conserved ➔ can be applied in seismic quantitative studies (e.g. estimation of amount of energy loss during wave propagation)